Game Theory and Strategy

Introduction

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Game Theory: Definition and Assumptions

- Game theory studies strategic interactions within a group of individuals
  - Actions of each individual have an effect on the outcome
  - Individuals are aware of that fact
- Individuals are rational
  - have well-defined objectives over the set of possible outcomes
  - implement the best available strategy to pursue them
- Rules of the game and rationality are common knowledge
Example

- 10 people go to a restaurant for dinner
- Order expensive or inexpensive fish?
  - Expensive fish: value = 18, price = 20
  - Inexpensive fish: value = 12, price = 10
- Everybody pays own bill
  - What do you do?
    - Single person decision problem
- Total bill is shared equally
  - What do you do?
    - It is a GAME
Example: A Single Person Decision Problem

Ali is an investor with $100

<table>
<thead>
<tr>
<th>State</th>
<th>Good</th>
<th>Bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Stocks</td>
<td>20%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Which one is better?
Probability of the good state $p$
Assume that Ali wants to maximize the amount of money he has at the end of the year.

Bonds: $110
Stocks: average (or expected) money holdings:

$$p \times 120 + (1 - p) \times 100 = 100 + 20 \times p$$

If $p > 1/2$ invest in stocks
If $p < 1/2$ invest in bonds
An Investment Game

- Ali again has two options for investing his $100:
  - invest in bonds
    - certain return of 10%
  - invest it in a risky venture
    - successful: return is 20%
    - failure: return is 0%
  - venture is successful if and only if total investment is at least $200

- There is one other potential investor in the venture (Beril) who is in the same situation as Ali

- They cannot communicate and have to make the investment decision without knowing the decisions of each other

<table>
<thead>
<tr>
<th></th>
<th>Beril</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bonds</td>
</tr>
<tr>
<td>Ali</td>
<td>110, 110</td>
</tr>
<tr>
<td>Venture</td>
<td>100, 110</td>
</tr>
</tbody>
</table>
Entry Game

- **Strategic (or Normal) Form Games**
  - used if players choose their strategies without knowing the choices of others

- **Extensive Form Games**
  - used if some players know what others have done when playing

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**Diagram**

![Game Tree Diagram]

- **Nodes**:
  - **Out**: 0, 4
  - **In**: 2, 2, -1, 0

- **Nodes**:
  - **P**: subdivisions

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Some players have private (and others have incomplete) information.

Ali is not certain about Beril’s preferences. He believes that she is

- Normal with probability $p$
- Crazy with probability $1 - p$

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<tr>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>110,110</td>
<td>110,100</td>
</tr>
<tr>
<td></td>
<td>100,110</td>
<td>120,120</td>
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</table>

<table>
<thead>
<tr>
<th>Beril Bond Venture</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>110,110</td>
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<tr>
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Normal ($p$)

Crazy ($1 - p$)
The Dating Game

- Ali takes Beril out on a date
- Beril wants to marry a smart guy but does not know whether Ali is smart
- She believes that he is smart with probability $1/3$
- Ali decides whether to be funny or quite
- Observing Ali’s demeanor, Beril decides what to do
Game Forms

- **Moves**
  - **Simultaneous**
  - **Sequential**

- **Information**
  - **Complete**
    - Strategic Form Games with Complete Information
    - Extensive Form Games with Complete Information
  - **Incomplete**
    - Strategic Form Games with Incomplete Information
    - Extensive Form Games with Incomplete Information

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Outline of the Course

1. Strategic Form Games
2. Dominant Strategy Equilibrium and Iterated Elimination of Dominated Actions
3. Nash Equilibrium: Theory
4. Nash Equilibrium: Applications
   4.1 Auctions
   4.2 Buyer-Seller Games
   4.3 Market Competition
   4.4 Electoral Competition
5. Mixed Strategy Equilibrium
6. Games with Incomplete Information and Bayesian Equilibrium
7. Auctions
8. Extensive Form Games: Theory
   8.1 Perfect Information Games and Backward Induction Equilibrium
   8.2 Imperfect Information Games and Subgame Perfect Equilibrium
9. Extensive Form Games: Applications
   9.1 Stackelberg Duopoly
   9.2 Bargaining
   9.3 Repeated Games
10. Extensive Form Games with Incomplete Information
    10.1 Perfect Bayesian Equilibrium
    10.2 Signaling Games