Economics of Information and Contracts
Screening: Simple Examples

Levent Koçkesen
Koç University

Screening

1 / 29

What is Screening?
- A contracting problem with Hidden Information
- Uninformed party (principal) offers contract to informed party (agent)
- Examples
  - Insurance
    - Insuree knows her risk, insurer does not
    - Insurer offers several packages with different premiums and deductibles
  - Finance
    - Borrower knows the risk of project, lender does not
    - Lender offers several packages with different interest rates and collateral requirements
  - Hiring
    - Applicants know their ability, employer does not
    - Employer offers different packages varying wages, bonuses, etc.
  - Pricing
    - Buyer knows her valuation of the product, seller does not
    - Seller offers different qualities at different prices, or quantity discounts

Screening by a Monopolist: A Very Simple Example
- You are a wine producer
- A fraction \(c\) of your customers are connoisseurs while the rest are novice
- You can produce high quality and/or low quality wine
- The following table gives the cost of producing the two qualities of wine as well as the willingness to pay for them by the two types of customers:

<table>
<thead>
<tr>
<th>Quality</th>
<th>Connoisseur</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Quality</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>High Quality</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

- You choose which qualities to produce and their prices to maximize your expected profit

Screening by a Monopolist
- Suppose you can tell your customer’s type
- What should you do?
- Produce both qualities
- Sell high quality to connoisseur at $30 and low quality to novice at $6
- Your profit is

\[
(30 - 10) \times c + (6 - 2) \times (1 - c) = 20c + 4(1 - c)
\]
- Maximizes the total surplus:
  - Extra benefit from high quality is bigger (smaller) than the extra cost for connoisseur (novice)
  - This is first degree (or perfect) price discrimination
  - It is great if you can implement it
  - But usually you cannot observe types
  - You may try to ask them their types. What would they tell you?
You cannot observe types

What happens if you offer high quality at $30 and low quality at $6?

Here are the net payoffs of the two types from purchasing the two qualities:

<table>
<thead>
<tr>
<th></th>
<th>Connoisseur</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Quality</td>
<td>10 - 6 = 4</td>
<td>6 - 6 = 0</td>
</tr>
<tr>
<td>High Quality</td>
<td>30 - 30 = 0</td>
<td>12 - 30 = -18</td>
</tr>
</tbody>
</table>

High type wants to buy low quality
Everybody will buy low quality
Your profit will be 6 - 2 = 4 only
Less than if you could observe types and perfectly discriminate
What is the best you can do in this case?

Offer only low quality

<table>
<thead>
<tr>
<th></th>
<th>Connoisseur</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Quality</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>High Quality</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

What price should you set?
If you set price at $6 both types buy
Your profit is 6 - 2 = 4
If you set price at $10 only type C buys
Your profit is c(10 - 2) = 8c
Therefore your best pricing policy and resulting profit is:

<table>
<thead>
<tr>
<th>Price</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>c ≤ 1/2</td>
<td>6</td>
</tr>
<tr>
<td>c &gt; 1/2</td>
<td>10</td>
</tr>
</tbody>
</table>

Offer only high quality

<table>
<thead>
<tr>
<th></th>
<th>Connoisseur</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Quality</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>High Quality</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

What price should you set?
If you set price at $12 both types buy
Your profit is 12 - 10 = 2
If you set price at $30 only type C buys
Your profit is c(30 - 10) = 20c
Therefore your best pricing policy and resulting profit is:

<table>
<thead>
<tr>
<th>Price</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>c ≤ 1/10</td>
<td>12</td>
</tr>
<tr>
<td>c &gt; 1/10</td>
<td>30</td>
</tr>
</tbody>
</table>
You offer both qualities

- You want C to buy high and N to buy low quality wine
- You want to choose \( P_H \) and \( P_L \) to maximize your expected profit
  \[ c(P_H - 10) + (1 - c)(P_L - 2) \]
  \[ (IC_C) \]
- C must prefer high to low quality
  \[ 30 - P_H \geq 10 - P_L \]
- N must prefer low to high quality
  \[ 6 - P_L \geq 12 - P_H \]

Called Incentive Compatibility (IC) constraints

- And you want them to prefer to buy
  \[ 30 - P_H \geq 0 \quad (IR_C) \]
  \[ 6 - P_L \geq 0 \quad (IR_N) \]

Called Individual Rationality (IR) (or participation) constraints

Solving the problem

1. \((IR_N)\) holds with equality: \( P_L = 6 \)
   
   Suppose instead that \( 6 - P_L > 0 \). Then
   
   \[ 30 - P_H \geq 10 - P_L > 6 - P_L \]
   
   - We could increase \( P_H \) and \( P_L \) by the same amount without violating IC constraints and increase our profit
   - This also shows that we can neglect \((IR_C)\)

2. \((IC_C)\) holds with equality: \( 30 - P_H = 10 - P_L \)
   
   Suppose instead that \( 30 - P_H > 10 - P_L \). Then
   
   \[ 30 - P_H > 10 - P_L > 6 - P_L = 0 \]
   
   - We could increase \( P_H \) without violating any constraints and increase our profit

3. Therefore, we have
   
   \[ P_L = 6, P_H = P_L + (30 - 10) = 30 - (10 - 6) = 26 \]

What is the best?

<table>
<thead>
<tr>
<th>( c )</th>
<th>Low Only</th>
<th>High Only</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c \leq 1/10 )</td>
<td>( 6 )</td>
<td>( 12 )</td>
<td>( 2 )</td>
</tr>
<tr>
<td>( 1/10 &lt; c \leq 1/2 )</td>
<td>( 6 )</td>
<td>( 4 )</td>
<td>( 30 )</td>
</tr>
<tr>
<td>( c &gt; 1/2 )</td>
<td>( 10 )</td>
<td>( 8c )</td>
<td>( 30 )</td>
</tr>
</tbody>
</table>

- Both is always better than Low only
- If \( c \leq 1/2 \) producing both is the best
- If \( c > 1/2 \) producing High only is the best

Getting surplus from Novice customers is costly

- You have to leave the Connoisseurs some surplus
- As the fraction of Connoisseurs increases the benefit decreases and the cost increases

- Therefore, if there are enough Connoisseurs around you don’t care much about getting the surplus from the Novice
Monopolistic Screening: A Little Less Simple Example

What happens if you can determine the level of quality of wine

- You are a wine producer
- Half the customers have High taste and half have Low taste
- If a customer buys $q$ quality wine and pays $t$ her payoff is
  - High taste $u_H = 3q - t$
  - Low taste $u_L = 2q - t$
- If she does not buy her payoff is zero
- Cost of producing quality $q$ is $q^2$
- You choose $q$ and $t$ to maximize your profit $t - q^2$

Choose $q_H, q_L \geq 0$ to maximize

\[ \frac{1}{2}(2q_L - q_L^2) + \frac{1}{2}(3q_H - q_H^2) \]

If at the solution $q_L, q_H > 0$, then the derivative of this function with respect to $q_L$ and $q_H$ must be zero:

\[ \frac{1}{2}(2 - 2q_L) = 0 \]
\[ \frac{1}{2}(3 - 2q_H) = 0 \]

which are solved as $q_L = 1$ and $q_H = 3/2$.

Therefore,

\[ t_L = 2q_L = 2 \text{ and } t_H = 3q_H = \frac{9}{2} \]

Your profit is

\[ \frac{1}{2}\left(\frac{9}{2} - \frac{9}{4}\right) + \frac{1}{2}(2 - 1) = \frac{13}{8} \]

If you cannot observe types

- What happens if you offer quality 3/2 at price 9/2 and quality 1 at price 2?
- High type’s payoff if she buys low quality is higher than buying high quality
  \[ 3 \times 1 - 2 = 1 > 0 \]
- Everybody will buy low quality
- Your profit will be at most $2 - 1 = 1$
- Less than if you could observe types and perfectly discriminate
- What is the best you can do in this case?
- Offer a high quality wine at high price and a low quality wine at low price so that
  - High taste customer buys the high quality wine
  - Low taste customer buys the low quality wine
  - You maximize your expected profit
You want to choose \((t_H, q_H)\) and \((t_L, q_L)\) to maximize

\[
\frac{1}{2}(t_H - q_H^2) + \frac{1}{2}(t_L - q_L^2)
\]

But you also want

- High taste customer buys the high quality wine
  \[
  3q_H - t_H \geq 3q_L - t_L \quad (IC_H)
  \]
- Low taste customer buys the low quality wine
  \[
  2q_L - t_L \geq 2q_H - t_H \quad (IC_L)
  \]

Called Incentive Compatibility (IC) constraints

- And you want them to buy
  \[
  3q_H - t_H \geq 0 \quad (IR_H)
  2q_L - t_L \geq 0 \quad (IR_L)
  \]

Called Individual Rationality (IR) constraints

Let’s first show that \(IR_L\) holds as equality. Suppose not and it is at \(l\). Then type \(H\)’s allocation must be in the shaded region. Say it is at \(h\). But then we can increase \(t_L, t_H\) by the same small amount without violating any constraints. This would increase profit.

We can represent preferences in \((q,t)\) plane by indifference curves. An indifference curve is a locus of \((q,t)\) that give same utility. Note that High type has steeper indifference curves. This is known as single crossing condition.

We can ignore \(IR_H\). Since \(IR_L\) holds as equality, type \(L\)’s allocation must be on \(U_L = 0\), say \(l\). \(IC_H\) implies that type \(H\)’s allocation must be below \(U_H\). But then \(IR_L\) automatically holds.
Now show that \( IC_H \) holds as equality. Suppose not and it is at \( h \). But then we can increase \( t_H \) by a small amount and increase profit.

Now show that \( q_H \geq q_L \). Suppose type \( L \)'s allocation is \( l \). Then type \( H \)'s allocation must be above \( U_L \) and below \( U_H \). Therefore, it must be to the right of \( l \) and between \( U_L \) and \( U_H \).

Finally, we can ignore \( IC_L \). This easily follows from the facts that \( IC_H \) holds as equality and \( q_H \geq q_L \).

Solving the problem

We can also show these by algebra

1. \( (IR_L) \) holds with equality. Suppose instead that \( 2q_L - t_L > 0 \). Then

\[
3q_H - t_H \geq 3q_L - t_L \geq 2q_L - t_L > 0
\]

- We could increase \( t_H \) and \( t_L \) by the same amount without violating \( IC \) constraints and increase our profit
- This also shows that we can neglect \( (IR_H) \)

2. \( (IC_H) \) holds with equality. Suppose instead that \( 3q_H - t_H > 3q_L - t_L \). Then

\[
3q_H - t_H > 3q_L - t_L \geq 2q_L - t_L = 0
\]

- We could increase \( t_H \) without violating any constraints and increase our profit
3. $q_H \geq q_L$. Because from $IC_L$ and $IC_H$ we have
$$2(q_H - q_L) \leq t_H - t_L \leq 3(q_H - q_L)$$

4. We can neglect $(IC_L)$ because
$$2(q_H - q_L) \leq 3(q_H - q_L) = t_H - t_L$$

Therefore, Low type’s individual rationality and High type’s incentive
compatibility constraints hold as equalities and we can ignore the
other two constraints

So, the problem becomes: Choose $(t_H, q_H)$ and $(t_L, q_L)$ to maximize
$$\frac{1}{2}(t_H - q_H^2) + \frac{1}{2}(t_L - q_L^2)$$

subject to
$$t_L = 2q_L$$
$$3q_H - t_H = 3q_L - t_L$$

Substituting the constraints into the profit function we get
$$\frac{1}{2}(3q_H - q_L - q_H^2) + \frac{1}{2}(2q_L - q_L^2)$$

The problem therefore becomes: Choose $q_L, q_H \geq 0$ to maximize
$$\frac{1}{2}(3q_H - q_L - q_H^2) + \frac{1}{2}(2q_L - q_L^2)$$

If the solution has $q_L, q_H > 0$, then first derivatives with respect to
$q_H$ and $q_L$ must be equal to zero
$$\frac{1}{2}(3 - 2q_H) = 0$$
$$\frac{1}{2}(-1) + \frac{1}{2}(2 - 2q_L) = 0$$

These are solved as
$$q_H = 3/2 \text{ and } q_L = 1/2$$

We can use the two constraints to solve for $t_L, t_H$:
$$t_L = 2q_L \Rightarrow t_L = 1$$
$$3q_H - t_H = 3q_L - t_L \Rightarrow t_H = 4$$

Optimal Wine Qualities

Your optimal quality/price choice:
- Quality 3/2 wine at price 4
- Quality 1/2 wine at price 1

Remember that if you could observe types you get
- Quality 3/2 wine at price 9/2
- Quality 1 wine at price 2

The problem with this was that high taste customer preferred buying the
low quality wine. What do you do to solve this problem?
- Keep giving him the same quality at lower price
  - This gives him positive surplus
- Reduce quality and price for the low taste customer
  - You do this to keep high type’s surplus at minimum
- Low taste customer was not a problem
  - Keep his surplus zero
Properties of the Solution

- Although this is a very simple example, several results carry over to more general models.
- Low type’s individuality constraint binds: she receives zero surplus.
- High type’s incentive compatibility constraint binds.
- As a result, High type obtains positive surplus.
  - Known as information rent.
  - You leave rent to High type to dissuade her from mimicking the Low type.
- High type consumes the efficient quantity.
  - This is the quantity that maximizes total surplus: revenue – cost.
    
    \[ \text{marginal revenue} = 3 = 2q_H = \text{marginal cost} \]

- Low type consumes less than the efficient amount.