KOÇ UNIVERSITY
College of Arts and Sciences
Department of Physics

Course: PHYS401 Quantum Mechanics I
Credits: 3
Semester: Fall 2003
Instructor: Professor **Tekin Dereli**

1. Midterm Exam: 5 November 2003, 15.30-16.45, Z42

Question: 1 Suppose a particle is described by the wave function

$$\psi(x,0) = \frac{N}{x^2 + a^2}.$$

i. Normalize $\psi(x,0)$. That is determine N in terms of a.
ii. Sketch $|\psi(x,0)|^2$ as a function of x.
iii. Where is the particle most likely to be found?
vi. What is the probability of finding the particle somewhere along the positive x-axis?
v. Calculate the expectation values $< x >$, $< x^2 >$ and $< p >$.

Question: 2 Consider a particle held in a one-dimensional, complex potential $V(x)(1+i\xi)$ where $V(x)$ is a real function of x and ξ is a real parameter.

Show that the probability density function $p(x,t) = |\psi(x,t)|^2$ and the probability current $j(x,t) = \frac{\hbar}{2mi}(\psi^* \frac{\partial}{\partial x}\psi - \frac{\partial}{\partial x}\psi^* \psi)$ satisfy the probability continuity equation

$$\frac{\partial}{\partial t}p + \frac{\partial}{\partial x}j = \frac{2\xi}{\hbar}V(x)p.$$

Question: 3 The quantum state of a simple harmonic oscillator at time $t = 0$ is given by the following superposition of stationary wave functions:

$$\psi(x,0) = Nu_1(x) + \frac{1}{\sqrt{2}}u_2(x) - \frac{1}{\sqrt{3}}u_3(x).$$

i. Find the constant N so that $\psi(x,0)$ is normalized. (Make use of the orthonormality of the stationary wave functions.)
ii. Determine $\psi(x,t)$ for any $t > 0$.
iii. If the energy E is measured, write down the possible outcomes of this measurement together with their corresponding probabilities.
iv. Calculate the expectation value $< E >$.