Time-Optimal Control of Automobile Test Drives with Gear Shifts

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joint work with

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Outline of the Talk

1. Introduction
2. Physical Model of the Car
3. A Mixed-Integer Optimal Control Approach
4. Test-driving Scenarios & Computational Results
5. Final Remarks
## Mixed-Integer Optimal Control (MIOC)

- Optimization of dynamic processes,
- Nonlinear stiff/non-stiff ODE/DAE models,
- Discrete and continuous controls,
- Nonlinear constraints.

## Tasks

- Reduce infinite-dimensional MIOCP to NLP.
- Want to avoid MINLP: How to treat discrete controls?

## Applications

- Chemistry, Bioinformatics, Engineering, Economics, ...
Introduction

Today’s Application

- Driver shall complete a prescribed track:
  - Time optimal, energy optimal, pareto, periodic, ...
- ODE model: Vehicle dynamics.
- Continuous decisions: Acceleration, brakes, steering wheel?
- Discrete decisions: When to select which gear?
- Constraints: Stay on track, control bounds, engine speed, ...

Christian Kirches, University of Heidelberg

Time-Optimal Automobile Test Drives with Gear Shifts
Forces

- \( F_{sf}, F_{lf}, F_{sr}, F_{lr} \): Side and lateral forces at front and rear tyre (Pacejka),
- \( F_{Ax}, F_{Ay} \): Accelerating forces attacking car’s c.o.g.

Coordinates

- \( x, y \) Global coordinate system,
- \( e_{SP} \) Displacement of car’s center of gravity,
- \( c_x, c_y \) Car body’s geometric center,
- \( \psi \) Angle of longitudinal axis against global ordinate.
Angles

- $\alpha_f$: Front wheel’s direction of movement against longitudinal axis,
- $\alpha_r$: Rear wheel’s direction of movement against longitudinal axis,
- $\beta$: Car’s direction of movement against longitudinal axis,
- $\delta$: Steering wheel angle against longitudinal axis.
Velocities

- \( v_f, v_r \) Front and rear wheel’s velocity into directions \( \alpha_f, \alpha_r \),
- \( \nu \) Car’s velocity into direction \( \beta \).
Controls

- $\dot{\delta}$ in $[-0.5, 0.5] \subset \mathbb{R}$: Time derivative of steering wheel angle,
- $\phi$ in $[0, 1] \subset \mathbb{R}$: Pedal position, translates to engine torque $M_{\text{eng}}$,
- $F_{\text{brk}}$ in $[0, 1.5 \cdot 10^4] \subset \mathbb{R}$: Braking force.

- $\mu$ in $\{1, 2, 3, 4, 5\} \subset \mathbb{Z}$: Selected gear, translates to gearbox transm. ratio $i^\mu_g$.

Model part relevant for $\mu$: Rear wheel drive

$$F^\mu_{lr} := \frac{i^\mu_g i_r}{R} M^\mu_{\text{eng}}(\phi, w^\mu_{\text{eng}}) - F_{\text{Br}} - F_{\text{Rr}},$$

$$M^\mu_{\text{eng}}(\phi, w^\mu_{\text{eng}}) := \text{some nonlinear function of } \phi \text{ and engine speed } w_{\text{eng}} \text{ in gear } \mu.$$
Optimal Control Problem

- ODE states trajectory $x(\cdot)$, control functions $u(\cdot)$,
- Free final time $t_f$ and global parameters $p$. 

Optimal Control Problem Class
### Discretization Grid

Select a partition of the time horizon \([t_0, t_f]\) into \(m - 1\) intervals

\[
t_0 < t_1 < \ldots < t_{m-1} < t_m = t_f.
\]

### Control Discretization

Select \(n_q\) base functions \(b_j : \mathbb{R} \rightarrow \mathbb{R}^{n_u}\). Using control parameters \(q \in \mathbb{R}^{n_q}\), let for all \(0 \leq i \leq m - 1\)

\[
u_i(t) := \sum_{j=1}^{n_q} q_{ij} b_{ij}(t) \quad \forall t \in [t_i, t_{i+1}]
\]

Choices: Piecewise constant/linear/cubic splines, continuity by external constraints.
Bock’s Direct Multiple Shooting Method: States

State Discretization

Introduce initial states $s_i$ for $0 \leq i \leq m - 1$ and solve $m$ IVPs

\[
\dot{x}_i(t) = f(t, x_i(t), q_i, p) \quad \forall t \in [t_i, t_{i+1}]
\]

\[
x_i(t_i) := s_i
\]

\[
s_{i+1} = x(t_{i+1}; t_i, s_i, q_i, p)
\]

Advantages

- Existence of solution of IVP,
- Improve condition of BVP,
- Distribute nonlinearity,
- Supply additional a-priori information using the $s_i$,
- Use state-of-the-art adaptive ODE/DAE solver with IND.
Bock’s Direct Multiple Shooting Method: Discrete NLP

Optimal Control NLP

\[
\begin{align*}
\min_{t_f, s_i, q_i, p} & \quad \phi(t_f, s_m, p) \\
\text{s.t.} & \quad \dot{x}_i(t) = f(t, x_i(t), q_i, p) \quad \forall t \in [t_i, t_{i+1}] \forall i \\
& \quad 0 = s_{i+1} - x_i(t_{i+1}; t_i, s_i, q_i, p) \quad \forall i \\
& \quad 0 = r^{eq}(t_0, x_0, q_0, \ldots, t_m, x_m, q_m, p) \\
& \quad 0 \leq r^{in}(t_0, x_0, q_0, \ldots, t_m, x_m, q_m, p)
\end{align*}
\]

\(x_i(t_{i+1}; t_i, s_i, q_i, p)\) denotes end point of solution of IVP \(i\) depending on initial values of \(t_i, s_i, q_i,\) and \(p.\)
Bock’s Direct Multiple Shooting Method: Solution of NLP

Exploiting Structure

- Partial separability of objective,
- Can evaluate intervals in parallel,
- Block sparse jacobians and Hessians,
- High-rank updates to Hessian (modified L-BFGS).

Solution of NLP by structured SQP method

- Reduce NLP to size of single shooting system,
- Dense active-set QP solvers: QPSOL, QPOPT, qpOASES, ...
Mixed-Integer Optimal Control Problem Class

Optimal Control Problem

\[
\begin{align*}
\min_{t_f, x(\cdot), u(\cdot), \omega(\cdot), p} & \quad \phi(t_f, x(t_f), p) \\
\text{s.t.} & \quad \dot{x}(t) = f(t, x(t), u(t), \omega(t), p) \quad \forall t \in [t_0, t_f] \\
& \quad 0 \leq c(t, x(t), u(t), \omega(t), p) \quad \forall t \in [t_0, t_f] \\
& \quad 0 = r^{eq}(t_1, x(t_1), \ldots, t_m, x(t_m), p) \\
& \quad 0 \leq r^{in}(t_1, x(t_1), \ldots, t_m, x(t_m), p) \\
& \quad u(t) \in U \subset \mathbb{R}^{n_u} \quad \forall t \in [t_0, t_f] \\
& \quad \omega(t) \in \Omega \subset \mathbb{R}^{n_\omega} \quad \forall t \in [t_0, t_f]
\end{align*}
\]

\[\Omega := \{\omega^1, \omega^2, \ldots, \omega^{n_\omega}\} \subset \mathbb{R}^{n_\omega} \text{ is a finite set of control choices, } |\Omega| < \infty.\]
Inner Convexification for Integer Controls

Let $\Omega$ be the finite set of all control choices.
Relax $\omega(t) \in \Omega$ to $w(t) \in \text{conv } \Omega \subset \mathbb{R}^{n_\omega}$.

Effects

+ Same number of controls $n_\omega$.
+ Dense QPs solvers faster, less active set changes.
  - Model must be evaluable & valid for potentially nonintegral $w(t)$.
  - How to reconstruct integral choice $\omega^*(t)$ from relaxed $w^*(t)$?
Outer Convexification for Integer Controls

Outer Convexification

For all $t$ and for each member $\omega^i \in \Omega \subset \mathbb{R}^{n_\omega}$ introduce $w_i(t) \in \{0, 1\}$. Let then

$$\omega(t) := \sum_{i=1}^{n_w} \omega^i w_i(t), \quad 1 = \sum_{i=1}^{n_w} w_i(t) \quad (SOS1)$$

Relax all $w_i(t) \in \{0, 1\}$ to $w_i(t) \in [0, 1] \subset \mathbb{R}$ to obtain choice $\omega(t)$.

Effects

- Increased number of controls $n_w = |\Omega|$ instead of $n_\omega$.
- Model can rely on integrality of the fixed evaluation points $\omega^i$.
- Relaxed solution often bang-bang in $w_i(t)$, thus integer.
- If not, SUR-0.5 as $\varepsilon$-approximative scheme.
Avoiding an Obstacle

- Start to the left, driving straight ahead at 10 km/h.
- Complete track in a time-optimal fashion.
- Predefined evasive manoeuvre to avoid obstacle.
Avoiding an Obstacle: Initialization

- Example: 40 multiple shooting nodes.
Avoiding an Obstacle: Solution

- Example: 40 multiple shooting nodes.
- Differential state trajectories:

![Differential State Function 0]
![Differential State Function 1]
![Differential State Function 2]
![Differential State Function 4]

- Control trajectories:

![Control Function 0]
![Control Function 3]
![Control Function 4]
![Control Function 5]
![Control Function 6]
Avoiding an Obstacle: Constraint Discretization

- Example: 10, 40, and 80 multiple shooting nodes.
Why is it integer?

- Maximum indicated engine torque depending on velocity.
Computation Times

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[Branch & Bound] [M. Gerdts, 2005] on a P-III 750 MHz

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[Outer Convexification] [K. et al., 2008] on an Athlon 2166 MHz
Racing on an ellipsoidal track

- Ellipsoidal track of 340m x 160m,
- Width of 5 car widths,
- Find time-optimal periodic solution.
Racing on an ellipsoidal track: Solution

Differential state trajectories:

Control trajectories:
Racing on an ellipsoidal track: Solution
Future Work

More complicated tasks
- More complicated circuits (think Istanbul Park, Hockenheimring, ...);
  requires slight modification of model & coordinate system.
- More detailed modelling of integer decision effects.

More sophisticated techniques
- For longer tracks: use a moving horizon optimization technique.
- Closed-loop offline optimization.
- Closed-loop online optimization with an industry partner.

Real-time Feasibility
- Computation for reasonable discretization quite fast.
- Active set QP can solve a sequence of related problems at cheap additional cost.
References


Thank you for your attention.

Questions?