Constrained and Distributed Optimal Control

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Challenges

Hybrid Control Design
&
Distributed Control for Large Scale Systems
Challenges

Hybrid Control Design

Switched Linear Systems
Constraint Satisfaction
At high level: Constrained Switched Linear System
External Switch Selects Mode of Operation
OAV Autonomous Flight

Objective
Follow given trajectories.
Waypoints= [Time,Space]

Model
Switched Linear – External Switch

Constraints
Speed and acceleration function of mode
Vehicle Dynamics Control

A driver aid for atypical road conditions, such as slippery, windy and bumpy roads

Nonlinear (Piece-wise linear) and Constrained System

- Traction Control (TC)
- Anti-lock Braking System (ABS)
- Electronic Stability Program (ESP)
- Active Front Steering (AFS) systems
- Active Suspension systems
- Active differential systems
Vehicle Dynamics Control

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Nonlinear (Piece-wise linear) and Constrained System
Integrated VDC via MPC

MIMO controller integrating local and global measurements coming from GPS, cameras, infrared and radar

- Front steering
- Four brakes
- Engine torque
- Active suspensions
- Active differential

Controlling Yaw, Roll, Pitch, Vertical, Lateral and Longitudinal Dynamics via Multiple Input

Enabling path following capabilities

Falcone, Kevizky, Borrelli from 2003 to today

- Longitudinal, lateral and vertical velocity/accelerations
- Yaw, roll and pitch angles/rates
- Position and velocity in a global frame

Davor Hrovat, Jahan Asgari, Eric Tseng, Mike Fodor

Ford Motor Company
Chameleon Visual Tracking

Objective

Tracking of a moving prey

Model

PTZ camera: Linear
Prey: Linear point mass

Constraints

Pan Tilt and Zoom constraints
Prey in tracking window $\forall$ unknown bounded accelerations
Common Problem Features

- **Objective**
  - Minimization of performance index
- **Models**
  - Linear, Uncertain
  - Switched-Linear, Uncertain
- **Constraints**
  - States and Inputs
Solved Problem ~ 40 years ago

- **Objective**
  - Minimization of performance index
- **Models**
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Focus of Research ~ 10 years ago

- **Objective**
  - Minimization of performance index

- **Models**
  - Linear, Uncertain
  - Switched-Linear, Uncertain

- **Constraints**
  - States and Inputs

Balluchi, Bemporad, Di Benedetto, Goodwin, Johansen, Johansson, Kerrigan, Maciejowski, Mayne, Morari, Pappas, Rantzer, Rawlings, Sangiovanni-Vincentelli, Sastry, Sontag, Tomlin, …. and many others.
Hybrid Constrained Optimal Control

\[
\min_U \sum_{k=0}^{N} ||Qx(k)||_p + ||Ru(k)||_p
\]

subject to

\[
x(k+1) = A_i x(k) + B_i u(k) + f_i \\
\text{if } [x(k), u(k)] \in X_i, \; i = 1, \ldots, s
\]

\[
Ex(k) + Lu(k) \leq M, \; k = 0, 1, 2, \ldots
\]

\[
x(k) \in \mathbb{R}^n \times \{0, 1\}^{n_b}, \; u(k) \in \mathbb{R}^m \times \{0, 1\}^{m_b}, \; U \triangleq \{u(0), u(1), u(2), \ldots\}
\]

- Understanding solution structures and properties
- Solution computational methods and tools
Hybrid Constrained Optimal Control

Borrelli from 1999 to 2004

\[
\min_U \sum_{k=0}^{N} ||Qx(k)||_p + ||Ru(k)||_p
\]

subj.to \( x(k+1) = A_ix(k) + B_iu(k) + f_i \)
\( \text{if } [x(k), u(k)] \in \mathcal{X}_i, \ i = 1, \ldots, s \)
\( Ex(k) + Lu(k) \leq M, \ k = 0, 1, 2, \ldots \)

\[ x(k) \in R^n \times \{0, 1\}^{n_b}, \ u(k) \in R^m \times \{0, 1\}^{m_b}, \ U \triangleq \{u(0), u(1), u(2), \ldots\} \]

- **Understanding solution structures and properties**
- **Solution computational methods and tools**
Characterization of the Solution \((p=1,2,\infty)\)

Borrelli et al, ACC, 2000
Borrelli et al, AUTOMATICA, 2005

The solution to the optimal control problem is a time varying PWA state feedback control law of the form

\[
 u^*(k)(x) = \begin{cases} 
 F_1(k)x + G_1(k) & \text{if } x \in CR_1(k) \\
 \vdots & \vdots \\
 F_R(k)x + G_R(k) & \text{if } x \in CR_R(k)
\end{cases}
\]

\(\{CR_i\}_{i=1}^{R}\) is a partition of the set of feasible states \(x(k)\).

- **p=1, p=\infty:**
  \[CR_i(k) \triangleq \{x : M_i(j, k)x \leq K_i(j, k)\}\]

- **p=2:**
  \[CR_i(k) \triangleq \{x : x'L_i(j, k)x + M_i(j, k)x \leq K_i(j, k)\}\]
Hybrid Constrained Optimal Control

\[
\min_U \sum_{k=0}^{N} \|Qx(k)\|_p + \|Ru(k)\|_p
\]

subj.to \[x(k+1) = A_i x(k) + B_i u(k) + f_i \]
\[\text{if } [x(k), u(k)] \in \mathcal{X}_i, \ i = 1, \ldots, s\]
\[Ex(k) + Lu(k) \leq M, \ k = 0, 1, 2, \ldots\]

\[x(k) \in \mathbb{R}^n \times \{0, 1\}^{n_b}, \ u(k) \in \mathbb{R}^m \times \{0, 1\}^{m_b}, \ U \triangleq \{u(0), u(1), u(2), \ldots\}\]

- Understanding solution structures and properties
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Computational Flow

Problem Setup → Invariant set computation

Reachability Analysis → Polyhedral set manipulation

Local parametric problems → Multiparametric LP/QP

Solution postprocessing → LMI and polyhedral set manipulation

$u^* = f_{PWA}(x)$

Dynamic Programming

Google: mpt toolbox

Borrelli et al, JOTA, 2003
Borrelli et al, AUTOMATICA, 2006
Baotic, Borrelli et al, SICON, 2007
Summary

Systematic Model-Based Control Design
MIMO, PWA, Constraints, Logics

- Piecewise affine state feedback control law
- Off-line computation:
  Automatic partitioning and control law synthesis
- On-line computation: Lookup Table Evaluation
- Extended to Min-Max Constrained Problems

Borrelli, Bemporad, Morari, TAC, 2003
MPC Algorithm

\[
\min_{U} J(U, x(0)) \triangleq \sum_{k=0}^{N-1} \| Q(x(k) - x_{\text{ref}}) \|_p + \| R(u(k) - u_{\text{ref}}) \|_p
\]

subj. to \[
\begin{align*}
\{ & x(k+1) = f(x(k), u(k)) \\
& u(k) \in U \\
& x(k) \in X \\
& x(0) = x(t)
\end{align*}
\]

At time t:
• Measure (or estimate) the current state \( x(t) \)
• Find the optimal input sequence \( U^* \triangleq \{ u^*(t), u^*(t+1), \ldots, u^*(t+N) \} \)
• Apply only \( u(t) = u^*(t) \), and discard \( u^*(t+1), u^*(t+2), \ldots \)

Repeat the same procedure at time \( t+1 \)
Important Issues in Model Predictive Control

Even assuming perfect model, no disturbances:

\[
\text{predicted open-loop trajectories} \neq \text{closed-loop trajectories}
\]

- Feasibility
  Optimization problem may become infeasible at some future time step.

- Stability
  Closed-loop stability is not guaranteed.

- Performance
  Goal: \( \min \sum_{i=0}^{\infty} L(x(k+i), u(k+i)) \)
  What is achieved by repeatedly minimizing \( \sum_{i=0}^{N-1} L(x(k+i), u(k+i)) \)
Feasibility and Stability Constraints

$$\min_{U} J(U, x(0)) \triangleq P(x(N)) + \sum_{k=0}^{N-1} \| Q x(k) \|_p + \| R u(k) \|_p$$

subj. to \[
\begin{cases}
  x(k + 1) = f(x(k), u(k)) \\
  u(k) \in U \\
  x(k) \in \mathcal{X} \\
  x(0) = x(t) \\
  x(N) \in \mathcal{X}_f
\end{cases}
\]

$X_f$ is an Invariant Set

$P(x)$ is a Control Lyapunov Function.
Chameleon Visual Tracking

Objective
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Min-Max Predictive Control

\[
J^*_j(x_j) = \min_{u_j} J_j(x_j, u_j)
\]

subj. to \[
\begin{cases}
\text{Model} \\
\text{Constraints}
\end{cases}
\]

\[
J_j(x_j, u_j) = \max_{v_j, w_j} \left( \|Qx_j\|_p + \|Ru_j\|_p + V^*(x_{j+1}) \right)
\]

Model: \[
x_{j+1} = A(w_j)x_j + B(w_j)u_j + Ev_j
\]

Uncertainty: Additive \( v_i \in \mathcal{V} \), Polytypic \( w_i \in \mathcal{W} \)

Constraints: \[
Fx_j + Gu_j \leq f \quad \text{For all } v_i \in \mathcal{V}, w_i \in \mathcal{W}
\]
Addressing Feasibility: Control Law Design

- **Smooth Pursuit Control**
- **Min-Max Predictive Control**
- **Saccade Control**
- **Minimum Time Predictive Control**

Graph showing:
- **y- Tracking Error**
- **x- Tracking Error**
- **Reach set**
- **Scanning Algorithm**
Robotic Chameleon Video

Avin, Borrelli et al., IROS, 2006

Explicit Min-Max MPC Solved at 50Hz
Vehicle Dynamics Control

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Traction Control Experiment

2000 Ford Focus, 2.0l 4-cyl Engine, 5-speed Manual Trans

Borrelli et al., IEEE TCST, 2006

"..Traction control on the V-6 test car was just right -- perhaps unique in all the industry…."
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Kevizky, Falcone, Borrelli from 2003 to today
Vehicle Model - 11 States, 6 Inputs

Inputs
\[ \delta_f \] Front steering angle
\[ F_b \] FL, FR, RL, RR brakes
\[ \tau \] Desired engine torque

States
\[ \dot{y} \] Lateral velocity
\[ \dot{x} \] Longitudinal velocity
\[ \psi \] Yaw angle
\[ \dot{\psi} \] Yaw rate
\[ \gamma \] Lateral position (I.F.)
\[ X \] Longitudinal position (I.F.)

\[ x = [y, \dot{y}, \dot{x}, \psi, \dot{\psi}, Y, X, \omega_{fl}, \omega_{fr}, \omega_{rl}, \omega_{rr}] \]
Pacejka Tire model

\[ F = f(\alpha, s, \mu, F_z) \]
Autonomous Vehicle Tests and Experimental setup

Objective
- Minimize angle and lateral distance deviations from reference trajectory
- Double lane change
- Driving on snow/ice, at different entry speeds

System
- Jaguar X-type
- dSpace rapid prototyping system equipped with a DS1005 processor board Sampling time: 50 ms
- Differential GPS, gyros, lateral accelerometers
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