Optimizing product assortment under customer-driven demand substitution

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ABSTRACT

The problem of product assortment and inventory planning under customer-driven demand substitution is analyzed and a mathematical model for this problem is provided in this paper. Realistic issues in a retail context such as supplier selection, shelf space constraints, and poor quality procurement are also taken into account. The performance of three modified models, one that neglects customers’ substitution behavior, another that excludes supplier selection decision, and one that ignores shelf space limitations, are analyzed separately with computational experiments. The results of the analysis demonstrate that neglecting customer-driven substitution or excluding supplier selection or ignoring shelf space limitations may lead to significantly inefficient assortments. The effects of demand variability and substitution cost on optimal assortment and supplier selection decisions as well as on the optimal revenue are also investigated. The main contribution of this paper is the development of a practical and flexible model to aid retailers in finding optimal assortments to maximize the expected profit.

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1. Introduction

In order to survive in a competitive environment and to establish a strong position in the market, retailers should be able to manage their operational activities efficiently while providing an adequate customer service-level. Activities such as store and inventory management, establishing relationships with the suppliers, ordering and purchasing of products all contribute to operational costs, while additional costs, or rather loss of revenue, may incur due to poor quality procurement and customer dissatisfaction. An important trade-off in finding the right product assortment is that increasing variety increases customer satisfaction but has a negative effect on operational costs. As a result, when a retailer decides on which suppliers to work with and what product assortment to carry, it is important to understand the expectations and the purchasing behaviors of customers. Research on this topic shows that customers are frequently willing to buy a different color, size or brand within a product category if their favorite variant is either not offered or is temporarily out of stock, rather than going home empty handed. This behavior is indicated by the term customer-driven demand substitution and causes the demand for the remaining product types to increase, affecting their optimal order quantities and the product assortment decisions. In selecting what products to offer, retailers are subject to store related constraints such as shelf and storage space limitations. Therefore, maximizing profit in the existence of these operational issues is a challenging problem for retailers as they seek to rationalize assortment and inventory decisions at the category level.

Product assortment, demand substitution, supplier selection, and inventory management have been studied extensively; yet, to the best of our knowledge, there is no previous work that considers all these aspects together in the literature. This paper provides a tool for retailers to determine the product assortment, which considers supplier selection and inventory management decisions in the presence of shelf space limitations and substitution behavior of customers. The proposed tool optimizes these inter-related decisions for each product category with the goal of maximizing the retailer’s expected profits, under cost and demand parameters to be estimated by the retailer over a time horizon. Specifically, we introduce a mixed-integer programming model for the joint problem in order to determine which product types should be ordered from the suppliers, as well as the optimal ordering quantities for the offered product types. The model finds an optimal policy that maximizes expected total profit over a planning horizon for which demand and customers’ substitution preferences can be forecasted. By solving this model with different parameter settings designed in our computational experiments, we analyze the importance of various aspects of the problem. We identify the effect of substitution on the product assortment and profit by varying substitution cost parameters. In addition, we show numerically that incorporating the supplier selection decision into the determination of product assortment may result in significantly increased profit, and furthermore, considering shelf space limitations in the decision process leads to more profitable assortments.

The rest of the paper is organized as follows. Section 2 provides the necessary background and literature on product assortment,
demand substitution, and supplier selection problems. The mathematical model for the problem with single-period, stochastic demand is presented in Section 3. An illustrative example is also presented in this section. In Section 4, the model is analyzed computationally. Conclusions are drawn in Section 5, where possible extensions to the proposed model are also discussed.

2. Literature review

Our focus in this paper is on product assortment under demand substitution, together with other relevant retail store management issues such as shelf space allocation and supplier selection decisions. In this section, we first review previous work on inventory management in the existence of both product assortment and demand substitution, specifically in the retailing context, and then mention some related papers from the supplier selection literature. Several researchers considered demand substitution together with product assortment and inventory management decisions for production systems [12,26], whereas our focus here is on retail operations. The main difference between substitution in retail settings and production systems is that demand substitution decisions cannot be directly controlled by retailers, while a supplier can direct the fulfillment of the demand for one product with the inventory of another product in production systems. One can consider two forms of demand substitution: in static or assortment-based substitution, a consumer might substitute when her favorite product is not in the assortment carried by the store, whereas in dynamic or stockout-based substitution, a consumer might substitute when her favorite product is stock-out at the moment of purchasing. Mahajan and van Ryzin provide a literature survey and an analysis on the studies related to the assortment-based and stockout-based consumer choice models [16]. Kök et al. [15] also present a recent review of assortment planning problems that covers industrial practice as well as the academic literature.

There is a significant amount of work on inventory management under demand substitution (see for example Parlar and Goyal [21], Noonan [19], Rajaram and Tang [25], Avsar and Baykal-Gursoy [1], Netessine and Rudi [18]). Here, we only review in detail the literature that addresses the joint problem of product assortment and demand substitution. In an early study, Pentico [22] develops a model to find the optimal assortment under downward substitution and stochastic demand, without considering stockout-based substitution. Later, Pentico proposes an EOQ model to formulate inventory costs [23,24]. In all of these three papers, it is shown that the problem can be approximated by using an efficient dynamic programming formulation. The product assortment problem was reconsidered in late 1990s by van Ryzin and Mahajan [28] as well as Smith and Agrawal [27]. Van Ryzin and Mahajan [28] study the assortment planning problem with a stochastic demand, single-period setting with the multinomial logit (MNL) consumer choice model. Their model allows assortment-based substitution, but does not consider stockout-based substitution. Later, Mahajan and van Ryzin [17] propose a stochastic sample-path optimization method for the same model under both assortment-based and multiple rounds of stockout-based substitution. However, resource constraints and supplier selection are not considered in these papers. Cachon et al. [6] extend the model proposed by van Ryzin and Mahajan [28] in the existence of consumer search costs and show that neglecting consumer search behavior leads to an assortment with less variety and lower expected profits. Bish and Mad- dah [3] consider the model of van Ryzin and Mahajan [28] but investigate pricing issues as well.

Smith and Agrawal [27] study the assortment planning problem with multi-period base-stock inventory models under both assortment-based and stockout-based substitution, but allowed for one substitution attempt only. They represent substitution with a general exogenous choice model specified by first-choice probabilities and a substitution matrix. They present an approximate solution approach by simplifying the problem using service-levels and showing that substitution results in bounds on individual demands for products. Kök and Fisher [14] also employ an exogenous probabilistic model of substitution and develop an estimation methodology for the substitution rates by leveraging data from stores with varying assortments. They propose an iterative optimization heuristic for the assortment planning and inventory problem with one-level, stockout-based substitution in the presence of constraints on shelf space, discrete maximum inventory levels and delivery lead times. Gaur and Honhon [10] model the consumer choice using a locational choice type model. In addition to analyzing the assortment-based-substitution case, they also propose approximations for the stockout-based substitution case. Finally, a recent paper by Fadiloglu et al. [9] investigate the assortment planning problem under an exogenous deterministic demand substitution model, focusing on shelf space allocation. In this study, ordering and supplier selection decisions are not taken into account.

In order to position the model in this paper, a classification of the above papers with respect to the assumptions on the type of consumer-choice model employed and the nature of demand substitution is provided in Table 1. The above papers model consumer choice in two broad categories: multinomial logit choice (MNL) and exogenous demand (see the review paper by Kök et al. [15] for details). In addition, even though all of these papers take assortment-based substitution into account, only some consider stockout-based substitution as well. To make this distinction, as in most other papers, we refer to the case with only assortment-based substitution as the static case whereas both assortment and stockout-based substitution is referred to as the dynamic case. In Table 1, the columns indicate the consumer-choice model employed (MNL, exogenous, or other) and the rows indicate the substitution assumption (static or dynamic).

According to the above classification, the model proposed in this paper is akin to those in [27,14]. We represent demand substitution by an exogenous and relatively simple model. Our model utilizes a representation of aggregate demand substitution behavior rather than individual consumer choice. On the substitution type, we take into account multiple levels of assortment- and stockout-based substitution. The resulting model is cruder than the one possible in [27,17,14] in this aspect but has the virtue of being much simpler to analyze and requires significantly less data. In particular, we handle the single-period stochastic demand case by a scenario-based approach for representing demand randomness. Hence, we do not need to make specific assumptions on the demand distribution of individual products or the category itself. Consequently, the resulting mixed-integer programming (MIP) model can be solved quickly as long as the number of demand scenarios is bounded reasonably. We take advantage of the simplicity and flexibility of our approach to integrate several important constraints and cost items that are relevant in a retail context.

Our model can easily incorporate realistic issues such as shelf space limitations and ordering quantity quotas for suppliers and

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<th>Substitution type</th>
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supplier selection decisions together with product assortment and inventory management. It may be important to plan these issues simultaneously and our approach provides a tool for carrying out such an analysis. Let us take the supplier selection problem as an example. While this problem has been studied extensively in the literature, most of the existing research on this subject does not consider inventory management of the products in the assortment as part of the problem. In reality, the ordering policy and supplier choice affects one another. For instance, if frequent ordering is necessary due to inventory management reasons (e.g. perishable inventory), a supplier with low unit price but high ordering cost might generate a higher total cost than a supplier with a high unit price and low order cost. As another example, when suppliers offer quantity discounts, the trade-off between savings in purchasing and inventory holding costs should be considered. There are some mathematical programming formulations for the supplier selection problem such as Bender et al. [2], Buffa and Jackson [5], and Degraeve et al. [7], However, none of these models considers demand substitution. The basic supplier selection decision is easily modeled in our approach and we present examples of the interaction of the supplier selection and assortment decisions.

To summarize, in the retailing sector, decisions on product assortment, category management, selection of suppliers, and inventory levels are closely-related with each other. However, in the open literature, to the best of our knowledge, multiple products, multi-level product substitution, inventory planning and supplier selection are not considered in an integrated model. In this paper, we formulate the multi-product inventory, product assortment and supplier selection problem with multi-level demand substitution under resource constraints with the objective of maximizing expected profit. We evaluate various scenarios by solving the proposed mixed-integer programming problem to optimality. Since the mixed-integer programming problem can be solved efficiently for realistic-sized problems by a commercial solver such as Cplex [13], the decision maker can easily modify the model and analyze different scenarios such as the introduction of a new brand into the market. In that sense, our work can be considered to be a first step in developing a tool to aid retailers in their decision making process.

3. Problem setting and the mathematical model

3.1. Problem setting: definitions and assumptions

We consider two types of important decisions that must be made by a retailer. On the tactical level, the retailer must choose which products to offer to its customers and which suppliers to work with. On the operational level, given the assortment and the suppliers, the retailer must choose how much to order from each product taking into consideration consumers' reaction to the assortment and the inventory availability. Traditionally, these two problems have been treated separately. Relatively recently, researchers have emphasized the benefits of making assortment and ordering decisions jointly. However, in addition to these, other operational or tactical factors such as which suppliers to work with, ordering quotas and shelf space limitations have an impact on the retailer's performance. The problem setting in this paper addresses all of these factors.

A single product category with several products is considered. These products can be purchased from several suppliers. It is assumed that each product is offered by a unique supplier. Often in retailing, a supplier provides a set of product variants in one or more related product categories that are targeted towards different market segments, and serve different consumer needs. Therefore, establishing a long-term relationship with a supplier may ease some operational activities for the retailer but still the retailer may prefer to establish only a limited number of such relationships.

As in most of the literature in joint assortment and inventory planning (see the review paper by Kök et al. [15]), we consider a single inventory cycle. If inventory cycles can be assumed to be similar as is the case when the demand and the prices are stationary, this also approximates the situation over a longer-period consisting of several cycles. Alternatively, if demand/price non-stationarity between periods is more significant than the in-period random fluctuation, then a multi-period model would be more appropriate as in Yücel et al. [29]. Here, in a typical ordering cycle, the sequence of events in the decision problem is assumed to be as follows. First, the suppliers declare their products and the order quantity quotas that they can supply. Next, the retailer selects which products and how much to order considering the substitution behavior of customers and the relevant costs, revenues and the demand distribution associated with each product.

A significant factor that may impact both assortment and operational decisions is the randomness in demand. We model random demand by a joint demand distribution for all products in the category which is represented by a collection of demand scenarios with given probabilities. This distribution may be obtained, for instance, by considering the aggregate demand for a given category and by splitting it into product demands using the market share estimates of each product. The assortment and the suppliers are selected to maximize the expected profit over all scenarios. For each demand scenario, the substitution quantities and the resulting ending inventory levels at the end of the period have to be determined to calculate the expected costs and revenues.

Another important factor in this setting is the consumer's reaction to what is available on the shelf. We assume that if a product is not available either because it is not in the assortment or because it is stocked-out, it is substituted by another product or a lost sale occurs with deterministic proportions. In this way, we do not attempt to model individual consumer choice decisions but employ an exogenous model capturing aggregate demand behavior of customers, as can be observed by the market share of each product within the category. The deterministic proportion assumption is standard for stockout-based substitution models (see Netessine and Rudi [18] and the references therein). These deterministic proportions represent the aggregate substitution rates from one product to another. In the simplest case, these rates might be obtained by market research. If retailer specific data is ample, they can be generated by the methodology proposed by Kök and Fisher [14]. Alternatively, in case of availability of point-of-sale data for analysis, the method proposed by Öztürk et al. [20] can be utilized to estimate these rates.

3.2. The model

The optimization model considers one product category in a retail store consisting of a set of products offered in the market, denoted by $P$ and a set of suppliers offering these products, denoted by $S$. Each product is supplied by exactly one supplier, whereas a supplier may supply more than one product. The model determines which products should be ordered from the suppliers and the optimal ordering quantities that maximize expected profit in the existence of fixed ordering costs placed per order, fixed costs of supplier selection due to costs of establishing relations with suppliers, purchasing costs, inventory holding costs, costs incurred as a result of poor quality products received, and customer substitution costs. The constraints of the model include shelf space limitations and ordering quantity quotas of the suppliers.
The following notation is used in the model:

**Parameters**

- \( w_{ik} \): proportion of customers whose preference is product \( k \) that substitute product \( k \) with product \( i \) (\( w_{ik} = 0 \));
- \( c_i \): unit cost of purchasing plus transportation for product \( i \);
- \( o_{ij} \): cost of ordering per order placed with supplier \( j \);
- \( s_{ij} \): cost of selecting supplier \( j \);
- \( d_{ib} \): random demand for product \( i \) under demand realization \( b \);
- \( Q_{ij} \): order quantity quota for product \( i \);
- \( S_S \): shelf space limitation quantity for product \( i \);
- \( a_{ib} \): amount of satisfied demand for product \( i \) under demand realization \( b \);
- \( x_{smi} \): amount of product \( i \) used to satisfy \( m \)th substitution from product \( i \) under demand realization \( b \) (\( m = 1, 2, \ldots, M \), where \( M \) is a constant).

**Variables**

- \( z_{ib} \): ending inventory position of product \( i \) under demand realization \( b \);
- \( x_i \): quantity of product \( i \) to be ordered;
- \( y_i \): 1, if product \( i \) is ordered; 0, otherwise;
- \( o_{ib} \): 1, if an order is placed with supplier \( j \); 0, otherwise;
- \( x_{0ib} \): amount of unsatisfied demand for product \( i \) under demand realization \( b \);
- \( x_{smb} \): amount of product \( i \) used to satisfy \( m \)th substitution from product \( k \) under demand realization \( b \) (\( m = 1, 2, \ldots, M \), where \( M \) is a constant).

**Model**

Max \( TP = TR - TCO - TCSS - TCP - TCI - TCPO - TCS \)

subject to

1. \( TR = \sum_{b=1}^{B} \sum_{i \in P} p_{ib} (y_{ib} + x_i - z_{ib}) \) (2)
2. \( TCO = \sum_{j \in J} o_{ij} o_{ib} \) (3)
3. \( TCSS = \sum_{j \in J} s_{ij} o_{ij} \) (4)
4. \( o_{ib} \geq a_{ib} y_{ib}, \forall i \in P, \forall b \in S \) (5)
5. \( TCP = \sum_{b=1}^{B} x_{ib} \) (6)
6. \( TCI = \sum_{b=1}^{B} \sum_{i \in P} (z_{0ib} + x_i + z_{ib}) x_{ib} \) (7)
7. \( TCPO = \sum_{b=1}^{B} p_{ib} q_{ib} x_{ib} \) (8)
8. \( TCS = \sum_{b=1}^{B} \sum_{m=1}^{M} \sum_{k \in K} s_{mk} b_{mk} x_{mkb} \) (9)
9. \( x_{0ib} + \sum_{b=1}^{B} \sum_{b=1}^{B} x_{mkb} = d_{ib}, \forall i \in P, \forall b = 1, \ldots, B \) (10)
10. \( x_{0ib} + \sum_{b=1}^{B} x_{mkb} + z_{ib} = z_{0ib} + x_i, \forall i \in P, \forall b = 1, \ldots, B \) (11)
11. \( x_{mkb} \leq (d_{ib} - x_{0ib}) w_{mk}, \forall i \in P \setminus \{i\}, \forall b = 1, \ldots, B \) (12)

The objective function \( TP \) defined in Eq. (1) maximizes the total expected profit. In this equation, \( TR \) denotes the expected revenue, \( TCO \) denotes the expected cost of ordering, \( TCSS \) denotes the expected cost of supplier selection, \( TCP \) denotes the expected cost of purchasing, \( TCI \) denotes the expected cost of inventory holding, \( TCPO \) denotes the expected cost of poor quality products, and \( TCS \) denotes the expected cost of substitution. The expected revenue is formulated in Eq. (2) and the expected cost of ordering is expressed in Eq. (3). Eqs. (4) and (5) provide the formulation for the expected cost of supplier selection, where selecting a supplier means at least one order is placed with that supplier. The expected cost of purchasing is expressed in Eq. (6) and the expected cost of inventory is formulated in Eq. (7), where expected inventory is calculated as the average of entering and leaving inventory levels for all demand realizations. Eq. (8) represents the formulation of the expected cost of poor quality products and the expected cost of substitution is given in Eq. (9). The demand for a product is the sum of first-choice demand satisfied and all substitutions from other products to that product and it is expressed in Eq. (10). Eq. (11) ensures that for each product, the sum of the beginning inventory level for that product and the quantity ordered for that product should be equal to the sum of substitutions to that product from any first-choice product including itself and the ending inventory level of that product. The substitution behavior of a customer is expressed in Eqs. (12) and (13), which use the substitution rate matrix \( W \). The amount of any level of substitution from product \( k \) to product \( i \) cannot be more than a certain proportion of the demand incoming to product \( k \) either as a first-choice demand or via substitution from other products. The value of this proportion is obtained by multiplying the substitution rates in matrix \( W \), which exist in that substitution chain. The substitution chain includes all the products that the customer tries to substitute from product \( k \) up to product \( i \). Substitution inequalities are written for each level of substitution. Because of the complexity associated with higher substitution levels, we provide only the first two of them here. Eq. (14) represents the shelf space constraints, Eq. (15) represents the ordering quantity quota limitations for suppliers, and Eqs. (16) and (17) represent integrality and nonnegativity constraints.

In the above formulation, \( B \) denotes the number of demand scenarios and \( b_{ib} \) denotes the probability of scenario \( b \) such that \( \sum_{b} b_{ib} = 1 \). The demand for product \( i \) under the demand scenario \( b \) is estimated as \( d_{ib} \). Here, note that the substitution and inventory variables need to be duplicated for each demand scenario, hence, the model size increases with the number of scenarios. However, these continuous variables do not contribute as much to the problem complexity as the binary variables related to decisions on which suppliers to work with and which products to order. In fact, the number of these binary variables would be limited in a realistic problem, as a typical category would most likely consist of at most 40–50 products supplied by at most 5–10 major suppliers. We have observed in our computational experiments that the model can be solved in less than a second with a commercial MIP solver (Cplex) under default settings when 10 products, 5 suppliers and 100 demand scenarios exist.

The objective function comprises several types of costs. The retailer may decide on which of these costs to include in a particular
application depending on the circumstances. We think that the inclusion of all of these costs into the model yields a preferred solution. The supplier selection cost is a control parameter that prevents working with too many suppliers. In this regard, this cost may include the single-period amortized cost of working with the supplier but also other supplier intangible preferences. Similarly, the substitution cost may be regarded as another control parameter. It represents the cost of the loss of goodwill of customers that may reflect on the retailer at a future time point as lost sales. Clearly, it is difficult to identify when and in what magnitude the loss of goodwill will incur an actual cost. For this reason, in our model we include a penalty cost per each substitution or lost sales realized, as in most inventory models. In line with their positioning strategy, the retailers would assess whether their customers expect to find a wide product assortment, or a high quality of service at their stores. Naturally, higher consumer expectations translate to higher substitution costs, depending also on the product category under consideration. For example, since customers are typically less likely to substitute their favorite personal care items, the substitution cost parameter in the model would need to be set to a high value for such categories.

Unlike most of the previous papers in the literature, our model also handles multi-level substitutions by assuming a multi-stage flow structure. We assume that under this structure the demand-product allocation is such that the expected profit is maximized. This is a reasonable approximation of reality where customers select substitutes among available products in order to minimize their implied substitution costs.

Fig. 1 explains the role of the substitution variables in Eqs. (10)–(13). For each product and demand scenario , there are three sets of arcs with corresponding decision variables in the figure: (i) first-choice demand incoming to product with quantity , (ii) substitution demand incoming to product , one for each level of substitution, with value for level , and (iii) Substituted demand outgoing from product , one for each level of substitution, with value for level . The dummy product corresponds to lost sale.

Retailers usually have some resource limitations, which are here collectively called shelf space limitations. In practice, the shelf space limitations might limit the total space that the stocked products in the category cover within the store, the number of products that the assortment contains, the number of suppliers that the assortment selects, and/or the number of SKUs (store keeping units) that the assortment contains. In our model, we limit the number of SKUs in the assortment. Implementation of other types of shelf space limitations can be handled with slight modifications in the model.

3.3. Illustrative example

In order to illustrate how this model can be used, we provide a small example with three products/brands: and ; and suppliers: and , where supplies product and supplies products and . The initial inventory is assumed to be zero. Product denotes lost sales and at most 3 levels of substitution exists . The substitution matrix is provided in Table 2. The information in the table states that, for instance, among the customers whose favorite product is , all select product as the second choice, and choose product or in the third choice.

The optimal assortment provided by the solution of this model is composed of with quantity and with quantity , implying that supplier is not selected. With this assortment, all of the demand for products and are satisfied, whereas 40% of demand for product is lost and 60% of demand for product is substituted.

3.3.1. The importance of customer substitution behavior

In order to analyze the importance of customer substitution behavior, we first exclude the cost of substitution (TCS) from the objective function and solve this modified model. Next, we calculate the TCS corresponding to the optimal solution of the modified model and subtract this quantity from the optimal objective value.

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**Fig. 1.** An illustration of the rate of the demand substitution variables in the model for demand scenario .
When the substitution behavior of customers is ignored, the modified model proposes an assortment that generates a total profit of 8333 compared to 10,825; thus resulting in 23% profit loss in this example.

### The importance of supplier selection

In order to analyze the importance of supplier selection, the cost of supplier selection (TCSS) is excluded from the objective function and the modified model is solved. Similar to the previous case, in order to obtain the profit of the modified model, TCSS of the optimal solution is subtracted from the optimal objective value of the modified model. When we ignore the supplier selection decision in this example, the model proposes an assortment which generates a total profit of 7965, thus resulting in 30% profit loss.

### The effect of shelf space limitations

Next, we analyze the effect of shelf space limitations. If the assortment is decided without considering shelf space limitations, the optimal ordering quantities might not fit into the reserved shelf space for that category. In this case, a logical strategy for the retailer is to distribute the limited shelf space proportionally among the products according to the product quantities of the assortment. The optimal solution might not fit into the reserved shelf space constraints. We observed that while the original model provides an optimal profit of 1833, the profit of the modified model is 353, resulting in 81% profit loss.

### The effect of demand variability

Continuing with the same example, we can consider several demand scenarios with given probabilities of occurrence. We introduce two demand scenarios as follows: In scenario 1, the individual demands for products P1, P2, and P3 are 3000, 4000, and 5000, respectively, as in the deterministic case, and in scenario 2, the demands are 2500, 4300, and 5200. Note that, the total demand in both of the scenarios are the same. The probability that scenario 1 occurs is 0.3, whereas the probability of realizing scenario 2 is 0.7. The optimal assortment provided by this model is composed of P1 with quantity 2930 and P2 with quantity 7350 and has a total expected profit of 7130. Compared to the optimal solution provided for the original example, we observe that in this case the system orders more and results in reduced profit. That is, although the total demand is the same for both scenarios, increased variability in demand leads to lower profit for the retailer. This observation on the effects of increasing demand variability will be generalized in Section 4.5.

### Experimental analysis

In our computational experiments we generated instances with 10 products (with the 11th product representing lost sales), and 5 suppliers. We assume that customers perform at most 3 levels of substitution \((M = 3)\). This is a reasonable value for the maximum substitution level since substitution rates tend to become very small at higher levels. Moreover, we assume the substitution cost is a linear function of the substitution level, \(m\). That is, we let \(s_m = SC_i \times m\), where \(SC_i\) denotes the first level substitution cost from product \(i\). The first level substitution cost for a product \(i\) is a linear function of its margin, \(mg_i\), and is calculated as \(SC_i = \theta \times mg_i\), where \(\theta > 0\) is a parameter to be set by the decision maker depending on the category under consideration and the assortment and service expectations of the consumers.

The supplier-product availability matrix, \(A\), which is common for all experiments, is given in Table 5. We assume that each product corresponds to a brand so that a product cannot be supplied by more than one supplier, but a supplier can supply more than one product/brand.

The parameter values are generated according to Table 6. We generated 100 random data sets according to the provided distributions. The average of investigated values over these 100 data sets are provided as test results. For each data set, we generated 100 demand scenarios, i.e., we set \(b\) to 100, and \(\beta_0\) to 0.01 for each \(b\). GAMS [4] with Cplex version 9.1 [13] is used as the computational environment. The experiments are performed on a workstation with a 1.7 GHz Pentium 4 processor and 1 GB RAM running under Windows XP Professional OS. Solving the MIP model for each data set with 100 scenarios, consisting of 10 products and 5 suppliers, takes less than a couple of seconds on this platform.

### The impact of changing substitution costs

Table 7 presents the changes in the total profit, revenue and operating costs as the substitution cost parameter, \(\theta\), is varied from 0 to 1 (thereby generating increasing costs of substitution). In addition to average expected revenue and all operating costs, the sum
of operating costs is given under the column SOC in this table. In Table 8, the average percentage of first-choice demand satisfied is given under the column %ds, the average percentage of lost sales is provided under the columns %ls, %s, %t, respectively, and the average percentage of sum of all levels of substitutions is provided under the column %subs, where %subs = %ls + %s + %t. Table 8 shows how these values change as substitution cost increases. Figs. 2 and 3 illustrate the change in the costs and demand satisfaction percentages with respect to $\theta$.

In Fig. 2, it is seen that as substitution costs increase, even though total demand increases, total profit decreases since the sum of operational costs increases more than the total revenue.

The optimal system carries a limited assortment and selects a subset of suppliers and favors substitution since it generates no cost when $\theta = 0$. As $\theta$ increases, the optimal system extends the assortment and increases the number of selected suppliers. Therefore, supplier selection costs increase as seen in Table 7. This shows that substitution costs alter ordering and supplier selection decisions. An increase in substitution and lost sales costs results in increased amounts of purchases, resulting in decreased amounts of substitutions as seen in Fig. 3, as well as increased inventory. Therefore, purchasing and inventory costs increase as seen in Table 7. This shows that substitution costs affect purchasing and inventory decisions as well.

4.2. The importance of substitution behavior

We compare the solution of two models, the original one and the one that neglects substitution behavior. In Table 9, the optimal profit of the original model is provided under the column [TP], the optimal profit of the model solved without substitution cost is given under the column [TP w/o sc], the difference between these values is provided under the column [Diff], and the percentage of the difference compared to the optimal total profit of the original model is given under the column [%Diff], where \([\%\text{Diff}]=100\times(\text{[TP]}-\text{[TP w/o sc]})/\text{[TP]}\). The results show that in substitution costs increases profit loss if substitution behavior of customers is neglected, resulting in assortments which pay more substitution cost as shown in Fig. 4.

4.3. The importance of supplier selection decision

In our experiments, excluding the supplier selection decision resulted in the loss of more than half of the profit as shown in Table 10. In this table, the optimal expected profit of the original model is
under the column [Diff.], and the percentage of the difference compared to the optimal expected profit of the original model is given under the column [%Diff.], where \[\%\text{Diff.} = 100 \times \left(\frac{\text{TP w/o ss}}{\text{TP}}\right)\]. The reason for the profit loss is that if supplier selection is not included in the product assortment decision, the system offers an assortment supplied by more suppliers. Thus, a higher supplier selection cost is incurred.

### 4.4. The importance of shelf space limitations

As shown in the illustrative example in Section 3.3, in order to analyze the importance of the shelf space limitation in this problem, we first introduced an effective shelf space limitation of 8000 for the category and compared the optimal profit of the original model and the profit of the assortment obtained by distributing the limited shelf space proportionally among the products according to the product quantities of the assortment, which is proposed by the model without shelf space constraints. We set \( \theta \) to 0.3 and generated 100 random data sets according to Table 6 when we look at the average expected profit over 100 data sets, we find that while the original model proposes an assortment with an average profit of 19,872, the model which excludes shelf space constraints has an average profit of 15,453. Therefore, excluding shelf space limitations results in 22.2% profit loss. When we examine the assortments proposed by both models, we observe that the model without shelf space limitation works with more suppliers paying more supplier selection costs. In order to generalize this observation we also performed experiments on the original model and observed the number of suppliers, which is denoted by [N(S)]. Table 11 shows the results obtained for the same experimental data except that the shelf space limitation varies. The results show that as shelf space limitations become looser, the number of selected suppliers increases though there exists a fixed cost for working with a supplier.

### 4.5. The impact of demand variability

In order to observe the impact of demand variability on substitution amounts as well as the optimal profit, ordering quantities, and supplier selection costs, we change demand variability and observe the corresponding parameters. We set \( \theta \) to 0.3 and generated 100 random data sets according to Table 6 except that the parameters of individual demands and prices of products are generated to 0.3 and generated 100 random data sets according to Table 6. When we look at the average expected profit over 100 data sets, we find that while the original model provides in Table 6. When we look at the average expected profit over 100 data sets, we find that while the original model provided 327,867, the assortment obtained by distributing the limited shelf space proportionally among the products according to the product quantities of the assortment, which is proposed by the model without shelf space constraints, has a total demand of 251,769. The results show that the model without shelf space limitation works with more suppliers paying more supplier selection costs. In order to generalize this observation we also performed experiments on the original model and observed the number of suppliers, which is denoted by [N(S)]. Table 11 shows the results obtained for the same experimental data except that the shelf space limitation varies. The results show that as shelf space limitations become looser, the number of selected suppliers increases though there exists a fixed cost for working with a supplier.

### Table 9

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>[TP]</th>
<th>[TP w/o ss]</th>
<th>[Diff.]</th>
<th>[%Diff.]</th>
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<td>199,711</td>
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<td>0</td>
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<tr>
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<td>175,364</td>
<td>9,830</td>
<td>5.2</td>
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<td>162,217</td>
<td>20,049</td>
<td>11.0</td>
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<td>181,089</td>
<td>108,835</td>
<td>72,254</td>
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<tr>
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</tr>
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### Table 10

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<th>[Diff.]</th>
<th>[%Diff.]</th>
</tr>
</thead>
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<td>95,015</td>
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### Table 11

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<th>Shelf space limitation (units)</th>
<th>[N(S)]</th>
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</thead>
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<tr>
<td>8,000</td>
<td>1.20</td>
</tr>
<tr>
<td>20,000</td>
<td>1.89</td>
</tr>
<tr>
<td>32,000</td>
<td>2.68</td>
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### Table 12

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<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
<td>( d_0 )</td>
<td>( d_0 = \sum_{i=1}^{N} x_i ), where ( \sum_{i=1}^{N} x_i = 1 ) and ( 0 \leq x_i \leq 1 ) for all ( i ) in ( P ) (total demand distribution varies from one experiment to another according to Table 13)</td>
</tr>
<tr>
<td>( p_i )</td>
<td>( p_i = \zeta_i + m_g ), where ( m_g ) is random with ( 4 \leq m_g \leq 8 ) for all ( i ) in ( P )</td>
</tr>
</tbody>
</table>
Table 13
The impact of demand variability

<table>
<thead>
<tr>
<th>Uniform distribution range</th>
<th>Total substitution percentage</th>
<th>Optimal profit</th>
<th>Cost of ordering</th>
<th>Cost of supplier selection</th>
<th>Cost of substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum d_i - 40.000 )</td>
<td>43.4%</td>
<td>103,259</td>
<td>108.2</td>
<td>81,414</td>
<td>33,254</td>
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<tr>
<td>39.000 ( \leq \sum d_i \leq 41.000 )</td>
<td>29.6%</td>
<td>81,034</td>
<td>156.4</td>
<td>102,344</td>
<td>26,840</td>
</tr>
<tr>
<td>35.000 ( \leq \sum d_i \leq 45.000 )</td>
<td>20.1%</td>
<td>66,309</td>
<td>203.5</td>
<td>102,344</td>
<td>22,397</td>
</tr>
<tr>
<td>30.000 ( \leq \sum d_i \leq 50.000 )</td>
<td>11.3%</td>
<td>55,183</td>
<td>289.2</td>
<td>125,198</td>
<td>19,172</td>
</tr>
</tbody>
</table>

Table 14
The impact of demand variability

<table>
<thead>
<tr>
<th>Demand variability increases</th>
<th>Original demand satisfied</th>
<th>Substitution percentages</th>
<th>The amount of lost sales</th>
<th>Ordering quantities</th>
<th>Optimal profit</th>
<th>Importance of substitution</th>
<th>Importance of supplier selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increases</td>
<td>Increases</td>
<td>Decrease</td>
<td>Decreases</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increases</td>
<td>Increases</td>
</tr>
</tbody>
</table>

used and for each data set, 100 demand scenarios are generated according to the provided demand distributions.

Table 13 provides the results of the experiments. Under the Total substitution percentage column, the sum of first, second, third levels of substitution percentages and lost sales percentages for all products is given. The substitution percentage of a product at a certain level is calculated as the ratio of the sum of substitutions from that product in that level to the demand for that product. Similarly, the lost sales percentage of a product is calculated as the ratio of lost sales amount from that product to the demand for that product.

It is observed in Table 13 that increasing demand variability decreases the substitution amounts. In addition, as the demand variability increases, the optimal profit decreases since the system orders more and pays more for ordering. By ordering more, the system makes less substitution and pays less substitution cost. However, since optimal profit decreases as demand becomes more variable, the proportion of substitution cost to total profit increases. Therefore, the importance of substitution increases as demand variability increases. Another result of increasing demand variability is increased supplier selection cost as observed in Table 13. In order to be able to satisfy the demand for products in case of variability in demand, the system extends its assortment and works with more suppliers. Therefore, as demand variability increases, the importance of integrated supplier selection decision increases. All of these observations are summarized in Table 14.

5. Conclusion

The problem of product assortment under customer-driven demand substitution in retail operations is analyzed in this paper. We developed a model for the multi-product inventory, product assortment and supplier selection problem with multi-level demand substitution. The behavior of the solution provided by the model is analyzed for the single-period problem with stochastic demand. The analysis is performed to examine the effects of three parameters, substitution cost, supplier selection cost, and shelf space limitations, separately. The results of the analysis can be summarized as follows. Varying levels of substitution costs affects purchasing, ordering, inventory management, and supplier selection decisions. In addition, high substitution and lost sales costs for retailers resulted in extended assortments and increased service-level, but reduced profit due to increase in operational costs. This information may be useful for retailers in positioning themselves in the market and for pricing decisions. When the effect of neglecting customers' substitution behavior in the model is analyzed, it is observed that ignoring substitution in product assortment decision results in reduced profit. Therefore, retailers should try to understand customer substitution behavior and incorporate it into their operational policies. We also observed that excluding supplier selection decision may lead to significant profit loss, hence retailers need also take into account the costs associated with it. When the effect of ignoring shelf space limitations is analyzed, it is concluded that considering shelf space limitations in the model results in more profitable assortment and retailers can work with more suppliers under more space. When the impact of variability on the profit, operational costs, and substitution amounts is analyzed, it is observed that as demand variability increases, retailers should order more in order to satisfy the demand and have to pay more operational costs. In addition, they might need to extend the width of their assortment by working with more suppliers. It is seen that as demand variability increases, substitution, and supplier selection decision become more important in product assortment decision.

To the best of our knowledge, the problem of multiple products, multi-level product substitution, inventory planning, and supplier selection in the existence of shelf space limitations has not been considered in an integrated model in the literature. Therefore, the main contribution of this study is to provide an efficient tool to determine the product assortment for retailers, which considers supplier selection and inventory management decisions in the existence of shelf space limitations and customers’ substitution behavior. In addition, using our model, retailers can position themselves in the market by solving the model for different customer segments that have different substitution behaviors. The model can also be used to decide on new product introduction, given the estimate of the profit margin of the product, substitution rates from/to this product, and its expected effect on the category demand. Another contribution of this paper is that the analysis of the developed model gives insights about the effect of substitution on product assortment, the importance of incorporating supplier selection decision to the product assortment problem, and the significance of shelf space limitations on determining the right assortment.

A number of modifications and extensions of the model developed in this paper are possible. For instance, in certain cases suppliers have to pay a fixed fee to the retailer in order to obtain shelf space in a given category. This situation can easily be handled by taking negative supplier selection costs. As for some extensions, first, vendor-managed inventory might be integrated into the model since it is an emerging trend in retailing that requires the rapid and accurate transfer of information between the retailer and its suppliers. Therefore, while integrating vendor-managed inventory into our model, inventory related issues should be modeled according to the agreements between the retailer and its suppliers. Second, promotional activities might be considered in the model. The studies show that promotions such as price cuts have significant effects on product choice [11] and product substitution [8].
However, it will require understanding the reaction of customers to price discounts and assessing the effect of brand loyalty.

### Acknowledgement

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### Appendix

See Table 15.

### References