Transforming 6-cycle systems into triple systems

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A *Steiner triple system* is a pair $(V, \mathcal{B})$ where V is a finite set and $\mathcal{B}$ is a collection of 3-element subsets of V called *triples* such that every 2-subset of V is contained in exactly one triple in $\mathcal{B}$. Similarly, a 6-*cycle system* of order v is a pair $(V, \mathcal{C})$ where V is a finite set and $\mathcal{C}$ is a collection of 6-cycles with vertices in V such that every edge of the complete graph on the set V is contained in exactly one 6-cycle in $\mathcal{C}$.

There are three different ways to transform a given 6-cycle (a, b, c, d, e, f) into two triangles:

- *inscribing* means to join pairs of vertices at distance two; in this way two inscribed triangles $\{a, c, e\}$ and $\{b, d, f\}$ are obtained
- *converting* means to delete two opposite edges $\{a, b\}$ and $\{d, e\}$ and replace them with the edges $\{a, e\}$, $\{b, d\}$
- *squashing* the 6-cycle means to identify its two opposite vertices a and d to get the *bowtie* $\{\{a, b, c\}, \{a, e, f\}\}$.

A complete answer to the question on the existence spectrum for a 6-cycle system having the property that its 6-cycles can be transformed (in three different ways) to produce triples of a Steiner triple system will be presented. Moreover, maximum packings and minimum coverings of complete graphs with 6-cycles that can be transformed to some partial triple systems will be discussed.