Covers and partial transversals of Latin squares.

Trent G. Marbach

Nankai-Baidu Joint Laboratory, Nankai University, China

The topic of transversals within Latin squares became of interest through the study of mutually orthogonal Latin squares (MOLS). The connection to such useful objects is strong, as a pair of Latin squares are a pair of MOLS if and only if each of the Latin squares decomposes into transversals. Along with this connection to MOLS, transversals have been of interest in the literature recently and have had a number of papers study them in their own right. It is well known that some Latin squares contain no transversals, but what is not so clear is how *close* a Latin square is to containing a transversal. Previously, this problem has been studied by studying partial transversals of Latin squares and observing how large we can make such a partial transversal. In this presentation, we will discuss an alternate substructure of a Latin square that can also be considered to be *close* to a transversal, which we call a cover.

A *cover* of a Latin square is a subset of entries of the Latin square such that each row, column, and symbol is represented at least once in the set of entries. We will present the results we have found regarding covers of minimum size within a Latin square, which shows a kind of duality between covers and partial transversals. We also will present work on minimal covers, which shows a clear distinctiveness between covers and partial transversals. After this, we will demonstrate a few other results of interest on this topic.

(Joint work with Darcy Best, Rebecca J. Stones, and Ian M. Wanless)