The Minimum Number of Vertices in Uniform Hypergraphs with Large Domination Number

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The *domination number* $\gamma(\mathcal{H})$ of a hypergraph $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H})$ is the minimum size of a subset $D \subseteq V(\mathcal{H})$ of the vertices such that for every $v \in V(\mathcal{H}) \setminus D$ there exist a vertex $d \in D$ and an edge $H \in E(\mathcal{H}$ with $v, d \in H$. We address the problem of finding the minimum number $n(k, \gamma)$ of vertices that a k-uniform hypergraph $\mathcal{H}$ can have if $\gamma(\mathcal{H}) \geq \gamma$ and $\mathcal{H}$ does not contain isolated vertices. We prove that

$$n(k, \gamma) = k + \Theta(k^{1-1/\gamma})$$

and also consider the s-wise dominating and the distance-l dominating version of the problem. In particular, we show that the minimum number $n_{dc}(k, \gamma, l)$ of vertices that a connected k-uniform hypergraph with distance-l domination number γ can have is roughly $\frac{k\gamma l}{2}$.

MSC2000: 05D05, 05C65, 05C69.

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