Designs Over Finite Fields

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In the last decade *designs over finite fields*, introduced in the seventies, have received a considerable attention in view of their applications in error-correction in randomized network coding. They generalize classical designs in terms of vector spaces as follows. A design with parameters t-(v,k,λ) over the finite field $\mathbb{F}_q$, briefly a t-$(v,k,\lambda)_q$ design, is a pair $(\mathcal{V},\mathcal{B})$ where $\mathcal{V}$ is the v-dimensional vector space over $\mathbb{F}_q$ and $\mathcal{B}$ is a collection of k-dimensional subspaces of $\mathcal{V}$ such that each t-dimensional subspace of $\mathcal{V}$ is contained in precisely λ members of $\mathcal{B}$.

In this talk I will give some insight into the problems we encountered as well as the results we obtained in determining necessary conditions on the existence of designs over finite fields with a prescribed automorphism group [1, 3].

Then I will focus my attention on the q-analogues of Steiner systems, that are designs over $\mathbb{F}_q$ having $\lambda = 1$. They are of particular interest because of their versatility; they indeed can be also viewed as (t,k)-spreads of $\mathrm{PG}(v-1,q)$ or perfect $(v,k,2k-2t+2)$-constant dimension codes. Finally, we briefly discuss possible automorphisms of the putative 2-$(7,3,1)_q$ design, the q-analogue of the Fano plane [2].

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2. M. Braun, M. Kiermaier, A. Nakić: On the automorphism group of a binary q-analog of the Fano plane, *European J. Combin.* **51** (2016), 443–457.

3. A. Nakić, M.O. Pavčević: Tactical decompositions of designs over finite fields, *Des. Codes Cryptogr.* **77** (2015), 49–60.

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