Near Butson-Hadamard Matrices with Small off-diagonal Entries

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(joint work with Oğuz Yayla)

For an integer $m \geq 2$ let ξ_m denote a primitive complex m-th root of unity. We call a v-periodic sequence $\underline{a} = (a_0, a_1, \ldots, a_{v-1}, \ldots)$ an m-ary sequence if $a_0, a_1, \ldots, a_{v-1} \in \mathcal{E}_m$, (where $\mathcal{E}_m = \{1, \xi_m, \xi_m^2, ..., \xi_m^{m-1}\}$), and an almost m-ary sequence if $a_0 = 0$ and $a_1, \ldots, a_{v-1} \in \mathcal{E}_m$. For $1 \leq t \leq v-1$ the autocorrelation function $C_{\underline{a}}(t)$ is defined by $C_{\underline{a}}(t) = \sum_{i=0}^{v-1} a_i \overline{a_{i+t}}$ where $\overline{a}$ is the complex conjugate of a. An m-ary or almost m-ary sequence $\underline{a}$ of period v is called a perfect sequence (PS) if $C_{\underline{a}}(t) = 0$ for all $1 \leq t \leq v-1$. Similarly, an almost m-ary sequence $\underline{a}$ of period v is called a nearly perfect sequence (NPS) of type $\gamma \in \{-1, +1\}$ if $C_{\underline{a}}(t) = \gamma$ for all $1 \leq t \leq v-1$. This definition is extended to any $\gamma \in \mathbb{R} \cap \mathbb{Z}[\xi_m]$ with small absolute value with respect to n. Such sequences can be used in applications requiring a sequence with good correlation properties. A NPS can be identified with a circulant near Butson-Hadamard matrix. A square matrix H of order v with entries in $\mathcal{E}_m$ is called a near Butson-Hadamard matrix $\mathrm{BH}_\gamma(v, m)$ of type γ if $H\overline{H}^T = (v-\gamma)I + \gamma J$ for a $\gamma \in \mathbb{R} \cap \mathbb{Z}[\xi_m]$. Very recently, new properties of m-ary $\mathrm{BH}_\gamma(v, m)$ matrices for $\gamma \in \mathbb{Z}$ are studied in Winterhof-Yayla-Ziegler (2014). In this work, we study m-ary BH_γ matrices for $\gamma \notin \mathbb{Z}$, and look for new BH_γ examples and their existence conditions. In addition, we use the methods in Winterhof et al to prove some nonexistence results for certain BH_γ matrices. We know that $\gamma \in \mathbb{R} \cap \mathbb{Z}[\xi_m]$. In this study, we consider the case $\mathbb{Z}[\xi_m] \backslash \mathbb{Z}$. Our motivation is to obtain BH_γ matrices having $|\gamma|$ as *small* as possible. For instance there exist BH_γ(3,7), BH_γ(4,7), BH_γ(5,5), BH_γ(6,5), BH_γ(7,5), BH_γ(7,7), BH_γ(8,5), BH_γ(9,5) for certain values of γ such that $\gamma \notin \mathbb{Z}$. In particular, BH_γ(5,5) exists for $\gamma \in \{-\xi_5^3 - \xi_5^2 + 2, 0, 5, \xi_5^3 + \xi_5^2 + 3\}$ with $|\gamma| \in \{1.38, 0, 5, 3.61\}$, respectively. The sequence $\underline{a} = (1, 1, -\xi_5^2, 1, 1)$ has $\gamma = -\xi_5^3 - \xi_5^2 + 2$ with $|\gamma| = 1.38$. We also consider BH_γ(8,5), it exists for $\gamma \in \{-\xi_5^3 - \xi_5^2 + 5, -\xi_5^3 - \xi_5^2, 8, \xi_5^3 + \xi_5^2 + 1, \xi_5^3 + \xi_5^2 + 6\}$ with $|\gamma| \in \{6.61, 1.61, 8, 0.61, 4.38\}$, respectively. The sequence $\underline{a} = (1, 1, \xi_5^2, \xi_5^3, 1, \xi_5^3, \xi_5, 1)$ has $\gamma = -\xi_5^3 - \xi_5^2 + 2$ with $|\gamma| = 0.61$. We obtained these examples by exhaustive computer

search by MAGMA. Moreover we present a method for excluding existence of BH_γ for certain dimensions in case $\mathbb{Z}[\xi_m]$ is not principal ideal domain that is an extension of a method presented in Winterhof et al.

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