Unbalanced Signed Unitary Cayley Graph

Abel A A Delgado

Universidade Federal de Rondônia, Brazil

`abel@unir.br`

A graph is signed if each edge is labeled as either positive or negative. A cycle is positive if it has an even number of negative edges. A signed graph (sigraph) is balanced if every cyle is positive. The first structure theorem says that a sigraph is balanced if the set of vertices can be partitioned into two subsets in such a way that edges labeled with positive sign have their endpoints in the same subset, and edges with negative sign joints vertices in differents subsets. Since most of signed graphs are unbalanced, Doreian and Mrvar proposed a method for establishing the partition structure of a signed relation for vertices that are close to exact balance as is possible. Denoting by N the total number of negative edges within partite sets and P the total number of positive edges between partite sets, the error of the partition is $P + N$. The signed unitary Cayley graphs $\mathcal{S}_n$ where $n = pq$ with p and q primes greater than 2 are unbalanced. For this signed graph we obtain a partition of the vertex set with partition error P ($N = 0$), where the number of negative edges between sets corresponds to the index of frustration. In addition, this configuration of vertices leads to define a quotient signed graph mod the ideal $\langle p \rangle$ over the ring $\mathbb{Z}_n/\langle p \rangle$, isomrphic to $\mathcal{S}_p$ which is a balanced sigraph.