Steiner triple systems without parallel classes

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A *Steiner triple system* of order v (or STS(v)) is a pair $(V, \mathcal{B})$ such that V is a v-set and $\mathcal{B}$ is a set of 3-subsets of V (called triples) such that each pair of elements of V occurs in exactly one triple in $\mathcal{B}$. Such a system exists if and only if $v \equiv 1$ or $3 \pmod 6$. A *parallel class* of an STS($6k+3$) is a set of $2k+1$ disjoint triples in the system, and an *almost parallel class* of an STS($6k+1$) is a set of $2k$ disjoint triples in the system. These can be thought of as analogous to matchings and near-matchings in graphs.

Despite significant efforts, few examples of Steiner triple systems without parallel classes or almost parallel classes are known. It is well known that there are Steiner triple systems of orders 15 and 21 without parallel classes, and Wilson showed that a sparse infinite family of STS($k+1$)s have no almost parallel classes. More recently, Darryn Bryant and I have constructed a second sparse infinite family of STS($6k+1$)s without almost parallel classes, and the first known infinite family of STS($6k+3$)s without parallel classes. In this talk I will review some of the history of this problem and outline the constructions underlying our results.

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