Orientation of Graphs with Constraints on the Out-degrees

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(joint work with M. Dalirrooyfard, K. Ehsani, K. Ozeki, R. Sherkati)

Let G be a graph and $F : V(G) \to 2^{\mathbb{N}}$ be a function. The graph G is said to be *F-avoiding* if there exists an orientation O of G such that $d^+_O(v) \notin F(v)$ for every $v \in V(G)$. In this talk using Combinatorial Nullstellensatz Theorem we show that if G is bipartite and it admits an orientation D such that $d^+_D(v) \geq |F(v)|$ for every vertex v, then G is F-avoiding. As a corollary, we find that if G is bipartite and $|F(v)| \leq \frac{d_G(v)}{2}$ for every $v \in V(G)$, then G is F-avoiding. The bound $|F(v)| \leq \frac{d_G(v)}{2}$ is best possible. For every graph G, we conjecture that if $|F(v)| \leq \frac{1}{2}\big(d_G(v) - 1\big)$ for every $v \in V(G)$, then G is F-avoiding.

Let k be an odd integer ($k \geq 3$). A mapping $\beta : V(G) \to \mathbb{Z}_k$ is called a $\mathbb{Z}_k$-boundary of G if $\sum_{v \in V(G)} \beta(v) \equiv 0 \pmod{k}$. Let β be a *$\mathbb{Z}_k$-boundary* of G. An orientation D of G is called a *β-orientation* if, for every vertex $v \in V(G)$, $d^+_D(v) - d^-_D(v) \equiv \beta(v) \pmod{k}$. Using some results on β-orientation of graphs, we show that the conjecture is almost true for the complete graphs.

MSC2000: 05C07, 05C20, 05C25, 05C31.

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