Hamiltonicity and Restricted Block-intersection Graphs of t-designs

Matt Walsh

Indiana-Purdue University Fort Wayne

`walshm@ipfw.edu`

(joint work with David Pike and Chip Vandell)

Given a combinatorial design $\mathcal{D}$ with block set $\mathcal{B}$, its traditional block-intersection graph $G_\mathcal{D}$ is the graph having vertex set $\mathcal{B}$ such that two vertices b_1 and b_2 are adjacent if and only if b_1 and b_2 have non-empty intersection. In this paper we consider the S-block-intersection graph, in which two vertices b_1 and b_2 are adjacent if and only if $|b_1 \cap b_2| \in S$. As our main result we prove that $\{1, 2, \ldots, t-1\}$-block-intersection graphs of t-designs with parameters $(v, t+1, \lambda)$ are Hamiltonian whenever $t \geq 3$ and $v \geq t+3$, except possibly when $(v, t) \in \{(8, 5), (7, 4), (7, 3), (6, 3)\}$.

MSC2000: 05B05, 05C45, 05C62.

Keywords: block designs, block-intersection graphs, Hamilton cycles.

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