A New Decomposition for k-ary Trees

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(joint work with K. Manes)

has either no children or k children (some of which might be empty).

A non-empty path, is a lattice path starting at the origin and consisting of upsteps u and downsteps d. A path that ends at the x-axis without ever falling below it is called Dyck path.

An ascent of a Dyck path is a maximal string of upsteps. It is well-known that every path P is uniquely determined by its sequence $(l_m)_{m\in[\mu]}$ of ascents by the formula $P = u^{l_1}du^{l_2}d\cdots u^{l_{\mu-1}}du^{l_\mu}$, where $u^j = uu\cdots u$ (j times).

For every forest $\mathcal{F}$ we denote by $P(\mathcal{F})$ the Dyck path with ascent sequence the sizes of the trees of $\mathcal{F}$.

In this context, it is shown that every non-empty k-ary tree T is decomposed to a forest $\mathcal{F}(T)$ of $(k-1)$-ary trees, such that $P(\mathcal{F})$ is a Dyck path.

Conversely, for every forest $\mathcal{F}$ of $(k-1)$-ary trees such that $P(\mathcal{F})$ is a Dyck path, there exists a unique k-ary tree T with the same size as $\mathcal{F}$, such that $\mathcal{F} = \mathcal{F}(T)$.

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