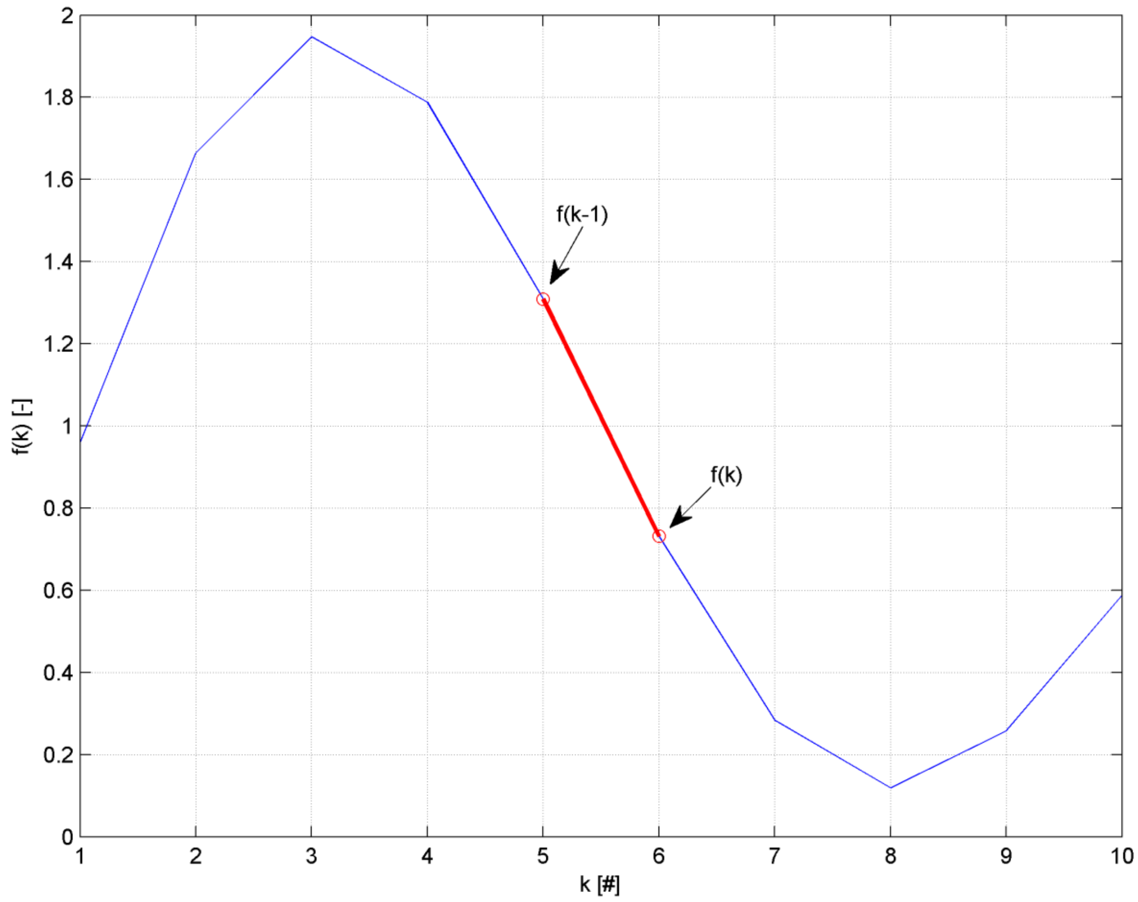
Discrete PID Controller

for use in Robotics Project #3

Outline

- Discrete Time Integrals and Derivatives
- Z-Transform
- Discrete TF of Integrals and Derivatives
- Discrete TF of PID
- Implementation of Digital PID
- Control Signal Limit (Saturation)

Discrete Time Integrals and Derivatives



Discrete Time Integrals and Derivatives (cont.)

- Discrete Time Derivative:

$$y(k) \approx \frac{f(k) - f(k-1)}{T_s}$$

Discrete Time Integral (Trapezoidal):

$$\int_0^{k*Ts} y(t)dt = y(k) \approx y(k-1) + \frac{f(k) + f(k-1)}{2} * T_s$$

z-Transform

- $$X[z] = \mathcal{Z}\{x[k]\} = \sum_{k=0}^{\infty} x[k] z^{-k}$$

$$z = A\, e^{j\theta} = A(\cos(\theta) + j\sin(\theta))$$

<u>Time shifting property:</u>

$$\mathcal{Z}\{x[k-n]\} = z^{-n}\, X[z], where,$$
$$\mathcal{Z}\{x[k]\} = X[z]$$

Discrete TF of Integrals and Derivatives

- **Discrete TF of Derivative:**

$$y(k) = \frac{f(k) - f(k-1)}{T_s}$$

$$Y[z] = \frac{F[z] - z^{-1}F[z]}{T_s}$$

$$\boxed{\frac{Y[z]}{F[z]} = \frac{z-1}{z\,T_s}}$$

Discrete TF of Integrals and Derivatives (cont.)

- **Discrete TF of Integral:**

$$y(k) = y(k-1) + \frac{f(k) + f(k-1)}{2}\, T_s$$

$$Y[z](1 - z^{-1}) = \frac{T_s}{2}\, F[z](1 + z^{-1})$$

$$\boxed{\frac{Y[z]}{F[z]} = \frac{T_s}{2}\frac{z+1}{z-1}}$$

Discrete TF of PID

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$$\frac{U[z]}{E[z]} = K_p + K_i \frac{T_s}{2} \frac{z+1}{z-1} + K_d \frac{z-1}{z\,T_s}$$

$$\frac{U[z]}{E[z]} = \frac{K_p(z^2 - z) + K_i \frac{T_s}{2}(z^2 + z) + \frac{K_d}{T_s}(z^2 - 2z + 1)}{z^2 - z}$$

$$\boxed{\frac{U[z]}{E[z]} = \frac{\left(K_p + K_i \frac{T_s}{2} + \frac{K_d}{T_s}\right) z^2 + \left(-K_p + K_i \frac{T_s}{2} - \frac{2K_d}{T_s}\right) z + \frac{K_d}{T_s}}{z^2 - z}}$$

Implementation of Digital PID

- $$\frac{U[z]}{E[z]} = \frac{\left(K_p + K_i \frac{T_s}{2} + \frac{K_d}{T_s}\right) z^2 + \left(-K_p + K_i \frac{T_s}{2} - \frac{2K_d}{T_s}\right) z + \frac{K_d}{T_s}}{z^2 - z}$$
  $$\frac{U[z]}{E[z]} = \frac{\left(K_p + K_i \frac{T_s}{2} + \frac{K_d}{T_s}\right) + \left(-K_p + K_i \frac{T_s}{2} - \frac{2K_d}{T_s}\right) z^{-1} + \frac{K_d}{T_s} z^{-2}}{1 - z^{-1}}$$
  $$U[z] = z^{-1} U[z] + a\, E[z] + b\, \mathrm{z}^{-1}\, \mathrm{E[z]} + \mathrm{c\, z}^{-2}\, \mathrm{E[z]}$$
  $$\boxed{u[k] = u[k-1] + a\, e[k] + b\, e[k-1] + c\, e[k-2]}$$

$$a = \left(K_p + K_i \frac{T_s}{2} + \frac{K_d}{T_s}\right)$$
$$b = \left(-K_p + K_i \frac{T_s}{2} - \frac{2K_d}{T_s}\right)$$
$$c = \frac{K_d}{T_s}$$

Control Signal Limit (Saturation)

Is control signal within limits ? If not, limit the control signal by the maximum value that can be achieved:

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if fabs(u) >= uMax
        u = u/fabs(u)*uMax;
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