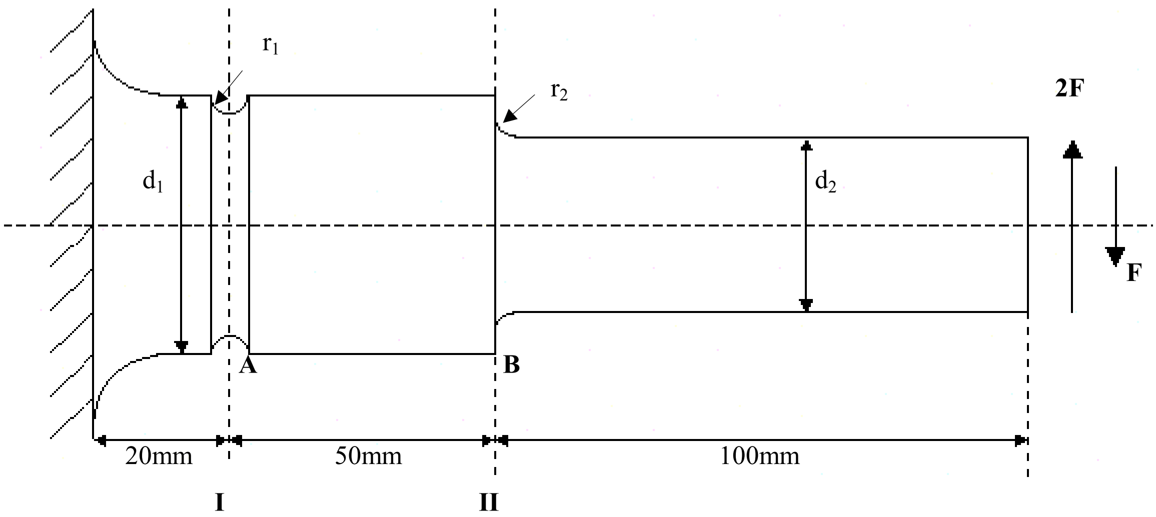
-Fatigue Design Criteria-

QUESTION 1) The circular beam shown in the figure is subjected to a fluctuating force. The direction and the magnitude of the force varies from 2F in positive y direction to F in negative y direction, in each cycle. The beam is cold rolled AISI 1030 Steel and the semi-circular groove and the fillet are machiced surfaces. The groove has a radius of r1=4mm and the fillet has radius of r2=3 mm. Find the maximum permissible limit of force F for an infinite life of operation, if the reliability and the factor of safety are 0.90 and 2 respectively. Use Modified Goodman Diagram in the failure analysis (Sut =520 MPa, Sy=440 MPa).



d_1=40 mm d_2=30 mm r_1=4 mm r_2=3 mm

Solution:

Critical sections of the beam are section-I, where the beam has a groove, and section-II, where the cross-section changes from 30 mm to 40 mm. Actually, the moment cretaed by the forces acting on the beam is maximum at the section where the beam is fixed to the wall. However, the beam is fixed to the wall with large radius fillet (which reduces the stress concentration at this point) and the ratio of the moment at this section to the moment at section-I is:

M / M_I = (170*F) / (150*F) = 1.13 which is small compared with the stress concentration factor at section-I.

During the application of force 2F, lowermost points of the beam are in tension. Since tensile stresses are more critical in failure analysis, points A and B will be checked (Type of Fatigue Loading ; Fluctuating).

Cold drawn AISI 1030 Steel : S_{ut}=520 MPa S_y=440 MPa

At point A

C_{load} = 1.0

$C_{surface}$ = A*(Sut)^b; A = 4.51, b = -0.265; see pp. 351

C_{size} = 1.189*(d^-0.097) = 0.86 see pp. 348

C_{size} = 0.85 (for $8 \leq d \leq$ 250 mm) reliability factor: $C_{reliab} = 0.897$ (0.9 reliability, pp. 353)

temperature factor : $C_{temp} = 1$ (not mentioned)

In order to find fatigue strength reduction factor we have to find fatigue stress concentration factor, K_f.

$K_f = 1 + q \cdot (K_t - 1)$

$q = 0.82$ (r=4 mm = 0.16 in, Sut = 520 MPa = 75 ksi; sqr(a) = 0.086 via linear interpolation; see Table 6-6, pp.362)

$K_t = 1.8$ (r/d = 4/32 = 0.125 , D/d = 40/32 = 1.25, figure E-5 , pp. 996; A = 0.944, b = -0.31)

$K_f = 1 + 0.82 \cdot (1.8 - 1) = 1.7$ (Assume that Kf*|Sigma_max| < Sy: hence Kfm = Kf: see Eq. 6-17, pp.385)

Assume that $K_{fm} = K_f = 1.7$ (you can later check that if this assumption is correct)

$S_e' = 0.5 \cdot S_{ut} = 0.5 \cdot 520 = 260$ MPa

$S_e = C_{load} \cdot C_{surf} \cdot C_{size} \cdot C_{temp} \cdot C_{rel} \cdot S_e'$ $= (1.0)\cdot(0.86)\cdot(0.85)\cdot(1)\cdot(0.897)\cdot(260) = 171$ MPa

The bending stress at point A is:

$$\sigma = \frac{M \cdot c}{I}$$

$M_{max} = 150 \cdot 2 \cdot F$ (N.mm) $M_{min} = -150 \cdot F$ (N.mm)

$d = 40 - 2 \cdot 4 = 32$ mm $I = \frac{\pi \cdot d^4}{64}$ $c = d / 2$

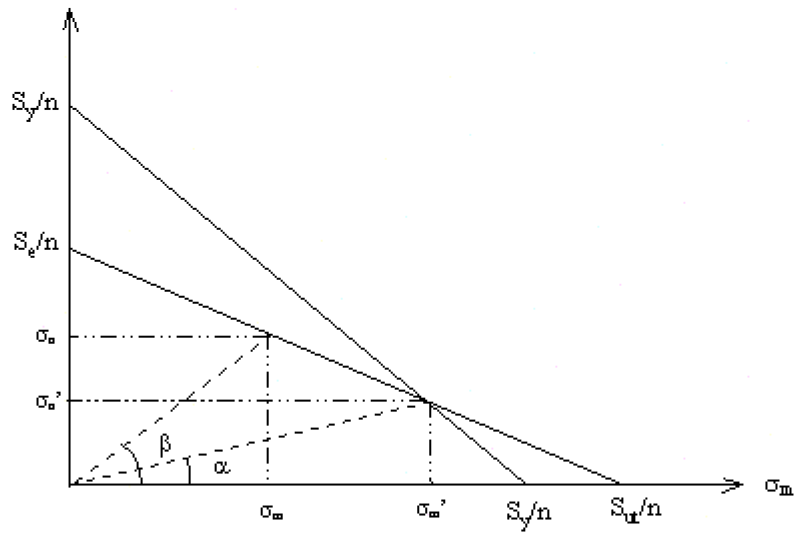
$\sigma_{min} = -0.047 \cdot F$ MPa

$\sigma_{max} = 0.093 \cdot F$ MPa

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$\sigma_m = (1.7)*0.023 \cdot F = 0.039F$ MPa $\sigma_a = (1.7)*0.070 \cdot F = 0.119F$ MPa

Using Modified Goodman Diagram:



Find the intersection point of Yield and Goodman lines:

solve;

$\sigma_a' + \sigma_m' = \sigma_y / n$

$\sigma_a' / S_e + \sigma_m' / S_{ut} = 1 / n$; where n=2

$\sigma_a' = 20$ MPa

$\sigma_m' = 200$ MPa

$\alpha = \tan^{-1}(\sigma_a' / \sigma_m') = 5.7°$

with the stresses that we have calculated, find the slope of load line:

$\beta = \tan^{-1} (\sigma_a / \sigma_m) = \tan^{-1} ((1.7*0.070) /(1.7* 0.023)) = 71.6°$

$\alpha<\beta$ which means;

The failure in this problem will be fatigue failure, so we will use Goodman line. If α was close to β we should check the stresses with yielding line.

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_{ut}}=\frac{1}{n} \qquad \frac{0.119\mathrm{F}}{171}+\frac{0.039\mathrm{F}}{520}=\frac{1}{2}$$

which gives F = 656.5 N

At point B

$C_{load} = 1$ $C_{surf} = 0.86$ $C_{reliab} = 1$ (same as point A)

$C_{size} = 1.189*(24)^\wedge(-0.097) = 0.87$

(for $8 \le d \le 250$)

$C_{temp} = 1.0$

q = 0.82 (Table 6-6)

$K_t = 1.61$ (r/d = 3/30 = 0.1 , d/D = 40/30 = 1.33 , A =~0.95, b = -0.23; figure E-2 , pp. 994)

$K_f = 1 + 0.82\cdot(1.61 - 1) = 1.5$

$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e' = 174$ MPa

The bending stress at point B is:

$$\sigma=\frac{M\cdot c}{I}$$

$M_{max} = 100\cdot 2\cdot F$ (N.mm) $M_{min} = -100\cdot F$ (N.mm)

d = 30 mm $I=\frac{\pi\cdot d^4}{64}$ c = d / 2

$\sigma_{min} = -0.038\cdot F$ MPa

$\sigma_{max} = 0.075\cdot F$ MPa

$$\sigma_m=\frac{\sigma_{max}+\sigma_{min}}{2} \qquad \sigma_a=\frac{\sigma_{max}-\sigma_{min}}{2}$$

$\sigma_m = 0.019\cdot F$ MPa $\sigma_a = 0.057\cdot F$ MPa

Using Modified Goodman Diagram:

with the stresses that we have calculated, the slope of load line:

$\beta = \tan^{-1}(\sigma_a / \sigma_m) = 71.6°$

$\alpha<\beta$

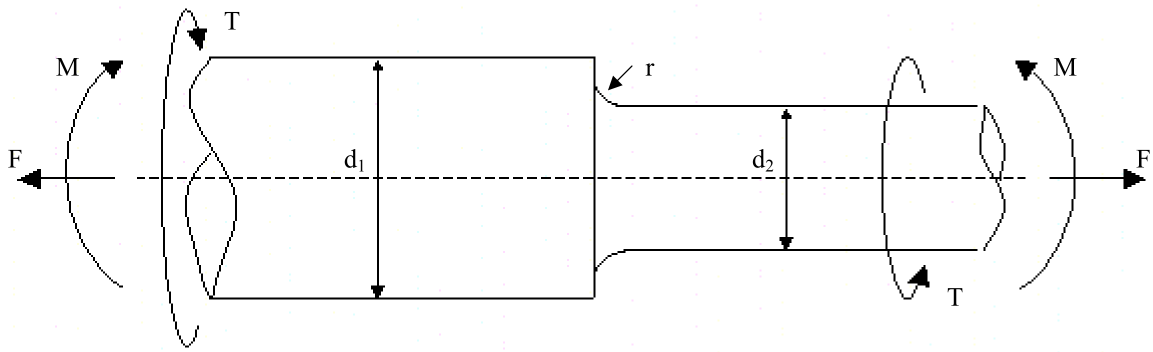
again we will use Goodman line: $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{0.086\,\mathrm{F}}{174} + \frac{0.029\,\mathrm{F}}{520} = \frac{1}{2}$$

which gives F = 935 N

So Point A is more critical and $F_{max} = 656.5$ N for infinite operation life.

QUESTION 2) A rotating circular shaft, machined from AISI 1095 Q&T steel, is subjected to a torque that varies from a value of 200 N.m to 400 N.m and to a fluctuating bending moment that varies from a value of -100 N.m to 300 N.m. Also an axial tensile force of 5 kN acts on the element. The torque and the bending moments have their peak values at the same time and their frequencies are same. Find the factor of safety of the element for an infinite operation life assuming the reliability as 0.999 . Use Modified Goodman Diagram (Material Properties; Sut = 1260 MPa, Sy=813 MPa).



$d_1 = 60$ mm $d_2 = 40$ mm $r = 8$ mm

Solution:

There is a combined loading in this problem. The load factor and stress concentration factor differs acoording to the type of loading. Therefore, they will used as load modifying factors instead of endurance limit modifying factors. They will be used for modifying the alternating components of the stresses (The critical section is at the fillet).

For the material: $S_{ut} = 1260$ MPa $S_y = 813$ MPa

C_{surf}= A*(Sut)^b = 0.76 (Machined surfaces, Table 6-6; A = 4.51, b = -0.265)

$C_{reliab} = 0.753$ (0.999 reliability, Table 6-4)

C_{size} = 1.189*d^(-0.097) = 1.189*(40^-0.097) = 0.83

$S_e = (0.76)\cdot(0.753)\cdot(Ctemp = 1).(0.83)\cdot(0.5)\cdot(1260) = 300$ MPa

Geometric properties:

r / d = 8 / 40 = 0.2 D / d = 60 / 40 =1.5

Axial Loading

C_{load}= 0.7 (for axial loading)

K_t = 1.55 (using figure E-1, pp. 994; the formula will return more accurate values!)

q = 0.96 (Sut = 1260 MPa = 0.145*1260 kpsi = 182.7 kpsi; From Table 6-6, we obtain sqrt(a) = 0.024 in; you can also use Fig 6-36)

$K_f = 1 + q\cdot(K_t - 1) = 1.53$

$k_e = 1 / (Cload * K_f) = 0.93$

axial load is constant, it will only create mean stress.

$\sigma_m = F/A$

F = 5000 N $A = \pi\cdot d_2^2 / 4$

$\sigma_m = 3.979$ MPa

Bending Moment

C_{load}= 1. 0

K_t = 1.4 figure E-2

q = 0.96

$K_f = 1+q\cdot(K_t - 1) = 1.38$

$k_e = 1 / K_f = 0.72$

bending moment is fluctuating. So it will cretae both mean and alternating stresses.

$\sigma = M\cdot c / I$

$I= \pi\cdot d_2^4 / 64$ $c= d_2 / 2$

M_{min} = -100000 N·mm M_{max} = 300000 N·mm

$M_m = (M_{max} + M_{min}) / 2$ = 100000 N.mm

$M_a= (M_{max} - M_{min}) / 2$ = 200000 N.mm

$\sigma_m = M_m \cdot c / I$ $\sigma_a = M_a \cdot c / I$

σ_m = 15.915 MPa σ_a = 31.831 MPa

modify σ_a with C_{load} and k_e

$\sigma_a = \sigma_a / C_{load}\cdot k_e$ = 44.2 MPa

Torsion

C_{load}= 1

K_t = 1.24 (using fig E-3)

q = 0.96

$K_f = 1+q\cdot(K_t - 1) = 1.23$

$k_e = 1 / K_f = 0.81$

Torsion is fluctuating. So it will cretae both mean and alternating shear stresses.

$\tau = T\cdot c / J$

$J= \pi\cdot d_2^4 / 32$ $c = d_2 / 2$

T_{min} = 200000 N·mm T_{max} = 400000 N·mm

$T_m = (T_{max} + T_{min}) / 2$ = 300000 N.mm

$T_a= (T_{max} - T_{min}) / 2$ = 100000 N.mm

$\tau_m = T_m \cdot c / J$ $\tau_a = T_a \cdot c / J$

τ_m = 23.873 MPa τ_a = 7.958 MPa

modify τ_a with C_{load} and k_e

$\tau_a = \tau_a / C_{load}\cdot k_e$ = 9.83 MPa

total results:

σ_m = 15.915 MPa + 3.979 MPa = 19.894 MPa (bending + axial force)

$\sigma_a = 44.2$ MPa

$\tau_m = 23.873$ MPa

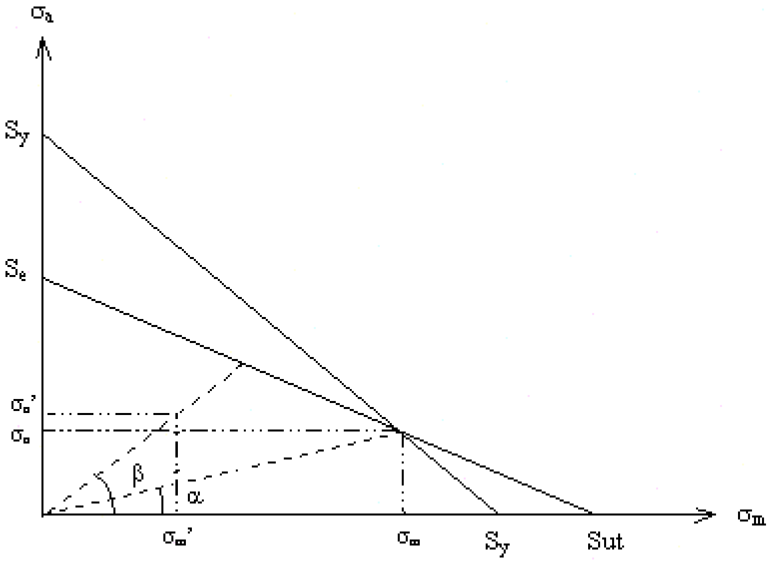
$\tau_a = 9.83$ MPa

According to Von-Misses, 2D Stress State:

$$\sigma_m' = \sqrt{\sigma_m^2 + 3 \cdot \tau_m^2} = \sqrt{(19.894)^2 + 3(23.873)^2} = 45.887 \text{ MPa}$$

$$\sigma_a' = \sqrt{\sigma_a^2 + 3 \cdot \tau_a^2} = \sqrt{(44.20)^2 + 3(9.83)^2} = 47.37 \text{ MPa}$$

slope of the load line: $\beta = \tan^{-1}(\sigma_a' / \sigma_m') = 46°$



To find the intersection point of Yield and Goodman lines, solve:

$\sigma_a + \sigma_m = \sigma_y$

$\sigma_a / S_e + \sigma_m / S_{ut} = 1$

$\sigma_a = 139.7$

$\sigma_m = 673.3$

$\alpha = \tan^{-1}(\sigma_a / \sigma_m) = 11.7°$

$\alpha<\beta$: Failure will be fatigue. Therefore, we will use Goodman line,

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{n}$$

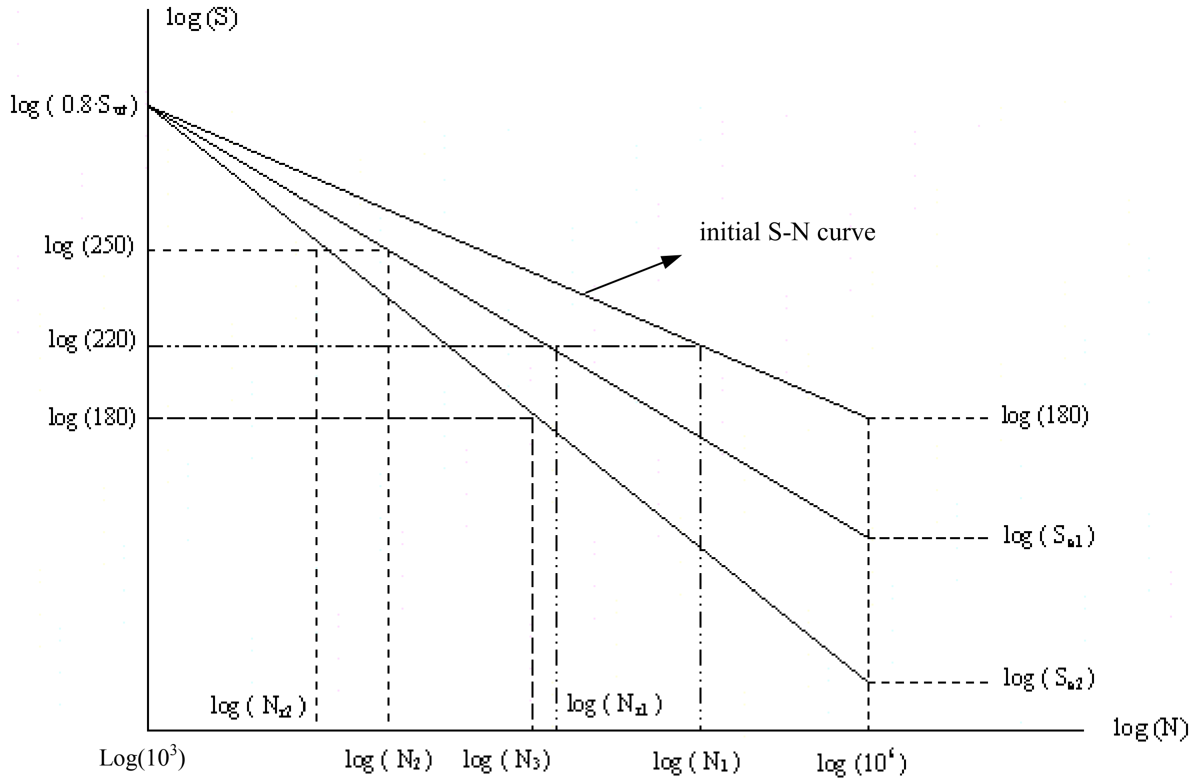
which gives n = 5.15

QUESTION 3) A machine part, made up of AISI 1060 Annealed Steel is first subjected to a completely reversed stress of magnitude 220 MPa, for 1000 days with 100cycles/day. Then the fully reversed loading is increased to 250 MPa and it's applied for 500 days with the same number of cycles per day.
Initial Endurance limit of the part is 180 MPa (Material Properties of AISI 1060; Sut = 625 MPa, Sy=372 MPa).

After these loadings:
i) What's the endurence limit of the part?
ii) For how many days it can be used under a fully reversed load of 180 MPa?

Use Manson's rule in the analysis.

Solution:
S_{ut} = 626 MPa S_y = 372 MPa S_e = 180 MPa



Total cycles for first loading: n_1 = 1000 days · 100 cycles/day = 100000 cycles

Corresponding lifetime (N_1) for 220 MPa loading in the initial S-N curve.

$$\frac{\log 220 - \log 180}{\log(0.8 \cdot 626) - \log 180} = \frac{6 - \log(N_1)}{6-3} \longrightarrow N_1 = 258028 \text{ cycles}$$

remaining life of the part for 220 MPa loading: $N_{r1} = N_1 - n_1$ = 258028-100000 =158028 cycles

The material is damaged after this loading. So, the S-N graph must be redrawn such that it will pass through the point log(158028), log(220) .

The new endurance limit is:

$$\frac{\log 500.8 - \log 220}{\log 500.8 - \log S_{e1}} = \frac{\log 158028 - 3}{6-3} \longrightarrow S_{e1} = 163 \text{ MPa}$$

Total cycles for second loading: $n_2 = 500$ days · 100 cycles/day = 50000 cycles
Corresponding lifetime (N_1) for 250 MPa loading in the modified S-N curve.

$$\frac{\log 250 - \log 163}{\log(500.8) - \log 163} = \frac{6 - \log(N_2)}{6 - 3} \longrightarrow \quad N_2 = 71951 \text{ cycles}$$

remaining life of the part for 250 MPa loading: $N_{r2} = N_2 - n_1$ = 71951-50000 =21951 cycles

The S-N graph must be modified again such that it will pass through the point log(21951), log(250) .

i) The final endurance limit is:

$$\frac{\log 500.8 - \log 250}{\log 500.8 - \log S_{e2}} = \frac{\log 21951 - 3}{6 - 3} \longrightarrow \quad S_{e2} = 106 \text{ MPa}$$

ii) Remaining lifetime N_3 for 180 MPa loading in the last modified S-N curve.

$$\frac{\log 180 - \log 106}{\log(500.8) - \log 106} = \frac{6 - \log(N_3)}{6 - 3} \longrightarrow \quad N_3 = 94570 \text{ cycles}$$

94570 cycles / 100 cycles per day = 945 days