Transverse Shear Stresses in Beams

SHEAR STRESSES IN BEAMS

In addition to the pure bending case, beams are often subjected to transverse loads which generate both bending moments *M*(*x*) and shear forces *V*(*x*) along the beam. The
bending moments cause bending normal stresses σ to arise through the depth of the beam, and the shear forces cause transverse shear-stress distribution through the beam cross section as shown in Fig. 1.

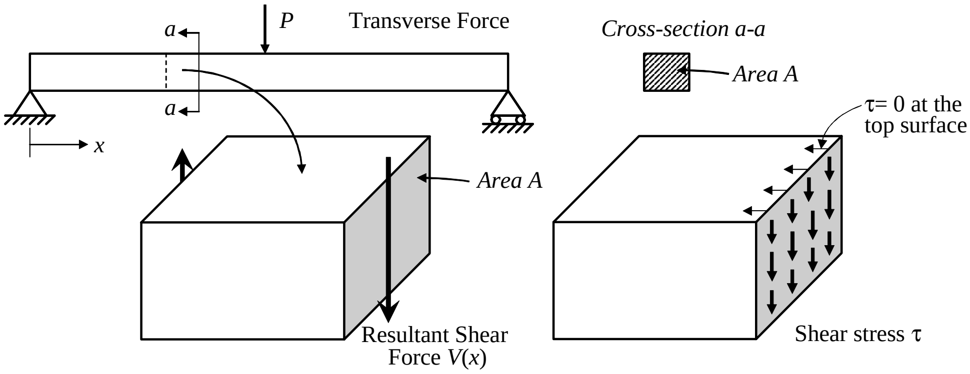


Fig. 1 Beam with transverse shear force showing the transverse shear stress developed by it

If we look at a typical beam section with a transverse stress as in Fig. 1, the top and bottom surfaces of the beam carries no longitudinal load, hence the shear stresses must be zero here. In other words, at top and bottom surfaces of beam section τ = 0. As a consequence of this, in determining the shear stress distribution, note the shear stress is **NOT EQUAL TO**:

$$\tau_{avg} = \frac{V(x)}{A} \qquad (1)$$

1 SHEAR FORMULA

Recall that in the development of the flexure formula, we assumed that the cross section *must remain plane and perpendicular* to the longitudinal axis of the beam after deformation. Although this is ***violated*** when the beam is subjected to both bending and shear, we can generally assume the cross-sectional warping described above is *small enough* so that it can be neglected. This assumption is particularly true for the most common cases of a ***slender*** beam, i.e. one that has a small depth compared with its length.
To determine the shear stress distribution equation, look at a loaded beam as Fig. 2:

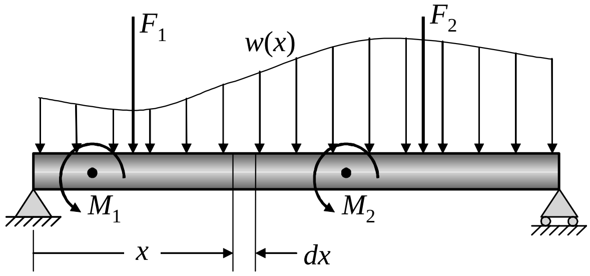


Fig. 2 Beam with applied loads

Look at a FBD of the element *dx* with the bending moment stress distribution only, Fig. 3, in which we do not need to look transverse forces if only horizontal equilibrium is considered.

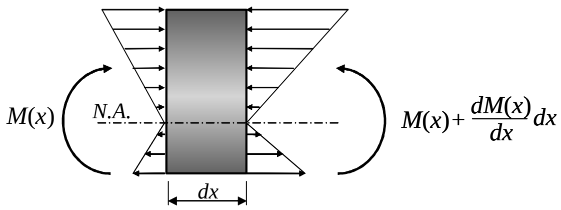


Fig. 3 Length of beam *dx* with normal stress distribution due to bending moment

Summing the forces horizontally on this infinitesimal element, the stresses due to the bending moments only form a couple, therefore the force resultant is equal to zero horizontally. Consider now a segment of this element a distance *y* above the *N.A.* up to the top of the element. In order for it to be in equilibrium, a shear stress τ_{xy} must be present, as shown in Fig. 4.

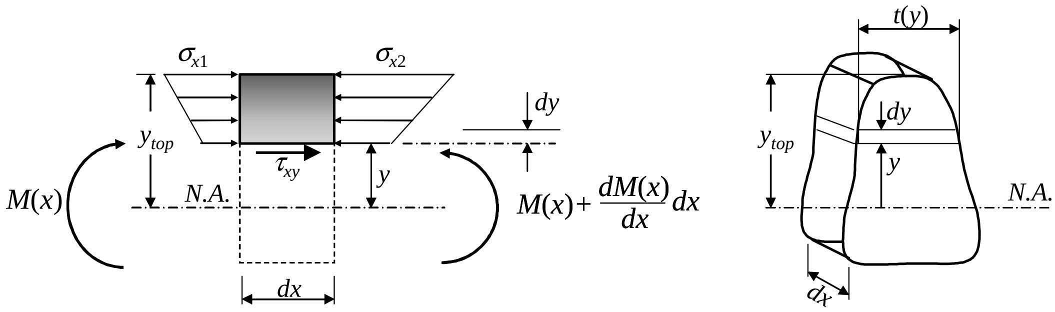


Fig. 4 Segment of length *dx* cut a distance *y* from *N.A.*, with equilibrating shear stress τ_{xy}

Let the width of the section at a distance *y* from the *N.A.* be a function of *y* and call it "$t(y)$". Applying the horizontal equilibrium equation, gives:

$$+\rightarrow \sum F_x = 0 = \int_y^{y_{top}} \sigma_{x1} t(y)dy - \int_y^{y_{top}} \sigma_{x2} t(y)dy + \tau_{xy} t(y)dx = 0 \qquad (2)$$

Substituting for the magnitude of the stresses using ETB gives:

$$\int_y^{y_{top}} \frac{M(x)y}{I} t(y)dy - \int_y^{y_{top}} \frac{(M(x)+dM(x))y}{I} t(y)dy + \tau_{xy} t(y)dx = 0$$

Simplifying and dividing by *dx* and $t(y)$ gives:

$$\tau_{xy} = \frac{dM(x)}{dx}\frac{1}{It(y)}\int_y^{y_{top}} yt(y)dy$$

But since $V(x) = \frac{dM(x)}{dx}$

then, the **Shear Stress Distribution** is given by:

$$\tau_{xy} = \frac{V(x)}{It(y)}\int_y^{y_{top}} yt(y)dy = \frac{V(x)Q(y)}{It(y)} = \frac{VQ}{It} \qquad (3)$$

where:

$V(x)$ the shear force carried by the section, found from the shear force diagram

I the second moment of area

$t(y)$ the sectional width at the distance *y* from the *N.A.*

$Q(y) = \int_y^{y_{top}} yt(y)dy = \bar{y}'A'$ A' is the top (or bottom) portion of the member's cross-sectional area, defined from the section where $t(y)$ is measured, and $\bar{y}'$ is the distance to the *centroid of* A', measured from the Neutral Axis.

2 SHEAR STRESSES IN BEAMS

Consider the beam to have a rectangular cross section of width *b* and height *h* as in Fig. 5

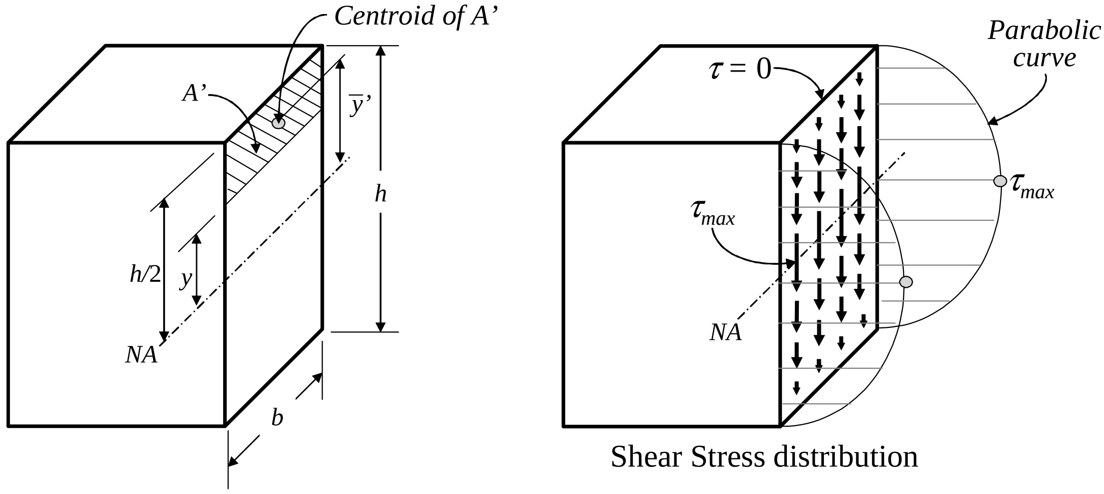


Fig. 5 Computation and distribution of shear stress in a rectangular beam

The distribution of the shear stress throughout the cross section due to a shear force *V* can be determined by computing the shear stress at an arbitrary height *y* from the Neutral Axis.

$$Q = \bar{y}' A' = \left(y + \frac{1}{2}\left(\frac{h}{2} - y \right) \right) \times \left(\left(\frac{h}{2} - y \right) b \right) = \frac{1}{2}\left(\frac{h^2}{4} - y^2 \right) b \qquad (4)$$

The second moment of entire area: $I = \dfrac{bh^3}{12}$

With $t = b$, applying the shear formula, Eq. (7.3), we have

$$\tau = \frac{VQ}{It} = \frac{V \times \frac{1}{2}\left(\frac{h^2}{4} - y^2 \right) b}{\frac{bh^3}{12} \times b} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2 \right) \qquad (5)$$

The result indicates that the ***shear stress distribution over the cross section is parabolic***, as plotted in Fig. 5. The shear force intensity varies from zero at the top and bottom, $y = \pm h/2$, to a ***maximum value at the neutral axis*** at $y = 0$

From Eq. (5), the maximum shear stress that occurs at the Neutral Axis is

$$\tau_{max} = 1.5 \frac{V}{A} \qquad (6)$$

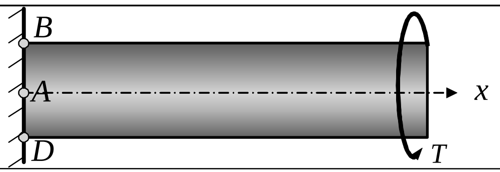
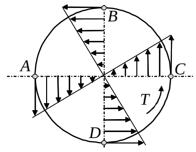
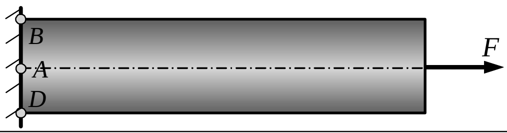
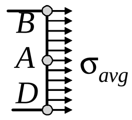
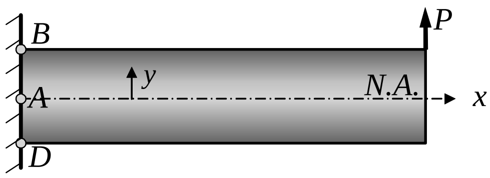
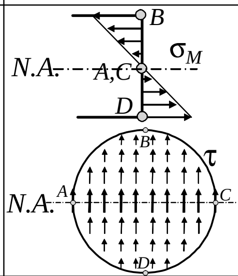
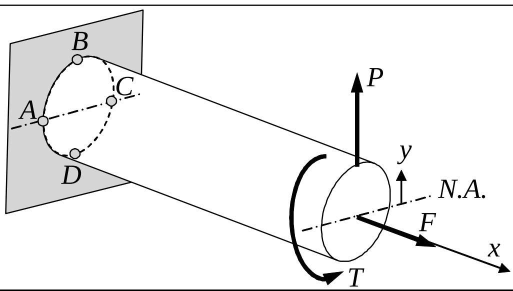
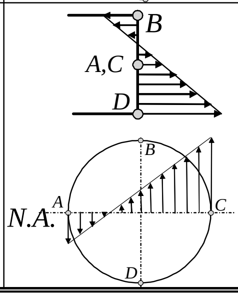
This same value for τ_{max} can be obtained directly from the shear formula $\tau = VQ/It$, by realizing that τ_{max} occurs where Q is largest. By inspection, Q will be a maximum when the area above (or below) the neutral axis is considered, that is $A' = bh/2$ and $\bar{y}' = h/4$.

By comparison, τ_{max} *is 50% greater than the average shear stress* determined from Eq. (1).

3 COMBINED LOADS

In the previous chapters, we developed methods for determining the stress distribution in a member subjected to different types of load such as an axial force or a transverse shear force, a torsional moment, and a bending moment. Most often,
the cross section of a member is subjected to several of these loadings *simultaneously*. As we shall see presently, we may combine the knowledge that we have acquired in the previous chapters. As long as the relationship between stress and the loads is ***linear*** and the geometry of the member would ***not undergo significant change*** when the loads are applied, the principle of superposition can be used. Here we are going to discuss
the situation due to tensile force F, torque T and transverse load P, as shown in Table 1.

Table 1 Superposition of individual loads

	Stresses Produced by Each Load Individually	**Stress Distributions**	**Stresses**
Torsional Load (Torque *T*)			Torsional shear stress $\tau_T = T\rho/J$
Axial Load (Force *F*)			Tensile average normal stress $\sigma_{avg}=F/A$
Bending Load (Transverse Force *P*)			Bending normal stress $\sigma_M = -My/I$ Transverse shear stress $\tau_V = VQ/It$
Combined Loads			Total normal stress $\sigma = F/A - My/I$ Total shear stress at *N.A.* $\tau = VQ/It \pm T\rho/J$