Discrete Complex Image Method With Automatic Order Selection

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***Abstract*—The discrete complex image method (DCIM) has been well-established and widely employed as a tool to compute spatial-domain Green's functions in planar multilayered media. Although the use of the DCIM in conjunction with the method of moments has been quite satisfactory for the efficient analysis of printed structures in layered media, it has not been considered much as a legitimate tool for commercial applications. Up until recently, there has been mainly two reasons for the reluctance of its acceptance in commercial products: the lack of a proper error criterion in the spatial domain and the manual selection of the order of images. As the former shortcoming has recently been eliminated by establishing a proper error criterion in the spatial domain, this paper intends to propose a viable solution for the latter problem, i.e., for the choice of the number of images. Note that, thus far, this choice has been based on the number of significant singular values obtained as a by-product of the method, which is neither rigorous, nor optimum in any sense, and renders it nonrobust and unreliable. As a remedy for this remaining shortcoming, a model selection approach known as minimum description length (MDL) has been introduced into the algorithm of the DCIM with success. The MDL approach, exploiting the regularities in the data, selects the order of the model based on the tradeoff between the quality of fitting and complexity of the model.**

***Index Terms*—Closed-form Green's functions, discrete complex image method (DCIM), layered media, minimum description length (MDL).**

I. Introduction

With the advent of high-speed computers, electromagnetic (EM) simulations of high-frequency systems and complex structures have been quite accessible by researchers and engineers, which undeniably furthers the boundaries of science and technology. In this regard, accurate and efficient computation of Green's functions plays an important role in computational electromagnetics and optics, as Green's functions are the key constituents of integral equations that govern the rules of EM wave propagation in different structures and media [1]–[3]. Since the solutions of integral equations are, in general, performed in the spatial domain using some numerical techniques, such as method of moments, finite-element methods, etc., computation of the spatial-domain Green's functions is crucial for the efficiency and accuracy of the numerical technique [4]–[10].

The spatial-domain Green's functions are usually obtained from their analytically expressible spectral-domain counterparts, and require the implementation of Hankel transform, which is also referred to as a Sommerfeld integral in the literature. In case of employing numerical quadrature in the implementation of the transformation, it is well known and well documented that the process becomes computationally expensive, mainly due to the oscillatory nature of the kernel of the transformation, rendering it almost impractical to use in conjunction with a numerical technique to solve the integral equations [2], [4]. As a viable and much more efficient alternative, analytical evaluation of the Hankel transforms of the exponential approximations of the spectral-domain Green's functions was proposed and referred to as the discrete complex image method (DCIM) [11], [12]. Upon its introduction with a rather restricted application domain, the method has been studied extensively, undergone a few major modifications and improvements [13]–[22], and motivated a few other alternatives of similar nature [23]–[28]. However, two major issues have lingered for a long time, which are: 1) the lack of a proper error metric incorporated in the DCIM algorithm that would ensure the end result in the spatial domain, which was only very recently resolved [29] and 2) an order selection algorithm, which will be the subject matter of this paper.

Order selection, in the context of the DCIM when used to obtain the closed-form Green's functions in the spatial domain, implies the selection of the number of exponential terms used in the approximation, which has been generally decided by the user, no matter which version of the DCIM is employed. Since the decision is usually based on the number of dominant singular values obtained as the by-product of the DCIM, which has been implemented either by a direct user intervention or by setting up a threshold for a measure of dominance of the singular values in the code, there is no underlying theory or practical evidence that this approach provides an optimum number of terms in any measure. Although, with the latest improvement of the DCIM [29], the spatial-domain error is guaranteed to be smaller in the weighted least squares sense as the number of terms increases, there is no guarantee that the choice of the number of the terms in the model is optimum. To remedy this shortcoming of the DCIM, a method known as the minimum description length (MDL) in the community of information theory has been adopted successfully [30]–[32]. The resulting DCIM, referred to as DCIM with model order selection throughout this paper, provides a choice of the model with the least code length, reflecting

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an optimal balance between the fitting error level versus model complexity. As an additional advantage of the method, this balance can be shifted by the user, if it is so desired, in favor of accuracy or computational cost, depending on the requirements of the problem at hand.

Since the proposed automatic order selection in the DCIM is based on a model-selection algorithm, it would be instructive to provide its brief introduction in Section II. In Section III, the MDL approach is then combined with the latest version of the DCIM, i.e., the one with the spatial error criterion, to propose a fully automated, robust, and accurate algorithm for obtaining the spatial-domain Green's functions in closed forms. It is followed, in Section IV, by some numerical examples and discussions to assess the robustness and accuracy of the proposed combination for some typical geometries. Finally, some conclusions are provided in Section V.

II. Model Selection

Model selection refers to finding the most suitable choice of the model, from a set of candidates, that provides the "best" description of the limited observations. The MDL approach, among various alternatives in the literature (e.g., [33]–[35]), stands out as probably the most popular approach for the model selection. According to the MDL philosophy, the most suitable model is the one that would lead to the most compression in the representation of the data. The rationale behind this view is the following: the more regularities in the data, the more we can compress the data. Therefore, the model that best captures the regularities in data leads to the most compression. Furthermore, the MDL approach does not assume the existence of a "true" model, but tries to find the most suitable candidate.

If the candidate set for the model selection consists of the same family of functions with different orders, e.g., polynomials or mixture of exponentials as in DCIM, then the corresponding problem is referred to as model order selection. In this approach, the observed data is typically structured as

$$y^{(n)} = f^{(r)}(x^{(n)}; \boldsymbol{\Theta}_r) + w^{(n)}, \qquad n = 1, \ldots, N \tag{1}$$

where

- $\{(x^{(n)}, y^{(n)}); n \in \{1, \ldots, N\}\}$ is the pair of data points for which a model describing the mapping from $\{x_n; n \in \{1, \ldots, N\}\}$ to $\{y_n; n \in \{1, \ldots, N\}\}$ is sought;
- $f^{(r)}(x; \boldsymbol{\Theta}_r)$ is the model function candidate of order r with parameters $\boldsymbol{\Theta}_r \in \mathcal{C}^m$;
- $w^{(n)}$ is the random noise sequence representing the fitting errors.

If the polynomial fitting problem is considered as an example, the model function corresponding to order r would have the form of

$$f^{(r)}(x; \boldsymbol{\Theta}_r) = c^{(0)} + c^{(1)}x + c^{(2)}x^2 + \cdots + c^{(r)}x^r \tag{2}$$

where the model parameter vector can be written as

$$\boldsymbol{\Theta}_r = \left[\, c^{(0)} \quad c^{(1)} \quad \ldots \quad c^{(r)} \,\right]^T \tag{3}$$

with $m = r + 1$. More relevant to the DCIM, if the exponential fitting problem is considered, the model function corresponding to order r would be given by

$$f^{(r)}(x; \boldsymbol{\Theta}_r) = \sum_{k=1}^{r} \beta^{(k)} e^{\alpha^{(k)} x} \tag{4}$$

where the corresponding model parameter vector is written as follows:

$$\boldsymbol{\Theta}_r = \left[\, \alpha^{(1)}\ \alpha^{(2)}\ \ldots\ \alpha^{(r)}\ \beta^{(1)}\ \beta^{(2)}\ \ldots\ \beta^{(r)} \,\right]^T \tag{5}$$

with $m = 2r$.

In the so-called two-step MDL approach [32], based on the assumption that $\{\mathcal{H}^{(k)}; k \in \mathcal{Z}_+\}$ represents the list of the candidate models (hypotheses) for the given data (in the DCIM case, $\mathcal{H}^{(k)}$ represents the mixture of k exponentials), the best model H would minimize the following:

$$L(D) = L(D|H) + L(H) \tag{6}$$

where $L(H)$ and $L(D|H)$ are the numbers of bits required to describe the hypothesis and the data based on the model, respectively, and $L(D)$ is the total number of bits to describe the data according to the chosen model.

Assuming independent and identically distributed (i.i.d.) Gaussian vector $[w^{(1)}\ w^{(2)}\ \ldots w^{(n)}]^T$, the conditional distribution for the data samples in (1) corresponding to hypothesis $\mathcal{H}^{(r)}$ can be written compactly as

$$p_{\mathbf{y}}(\mathbf{y}|\mathbf{x}; \boldsymbol{\Theta}_r, \sigma) = \frac{1}{(2\pi)^{N/2}\sigma^N} e^{-\|\mathbf{y} - \mathbf{f}^{(r)}(\mathbf{x}; \boldsymbol{\Theta}_r)\|_2^2 / 2\sigma^2} \tag{7}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the data sample vectors, $\mathbf{f}^{(r)}(\mathbf{x}; \boldsymbol{\Theta}_r)$ is the model vector, and $\|\mathbf{a}\|_2 = \sqrt{\mathbf{a}^* \mathbf{a}}$ corresponds to the vector 2-norm for any $\mathbf{a} \in \mathcal{C}^N$. The length of the Shannon–Fano code [36] corresponding to this distribution is given by

$$L(D|\mathcal{H}^{(r)}; \boldsymbol{\Theta}_r, \sigma) = -\log(p_{\mathbf{y}}(\mathbf{y}|\mathbf{x}; \boldsymbol{\Theta}_r, \sigma)) \tag{8}$$

which is a function of the model vector $\boldsymbol{\Theta}^{(r)}$, as well as the fitting error parameter σ. Note that the length expression in (8) corresponds to the negative log-likelihood function [37], and it can be further simplified to

$$L(D|\mathcal{H}^{(r)}; \boldsymbol{\Theta}_r, \sigma) = \frac{N}{2}\log(2\pi) + N\log(\sigma) + \frac{1}{2\sigma^2}\|\mathbf{y} - \mathbf{f}^{(r)}(\mathbf{x}; \boldsymbol{\Theta}_r)\|_2^2 \tag{9}$$

where the fitting error parameter σ can be treated either as an unknown, in which case it needs to be considered jointly with the model parameters $\boldsymbol{\Theta}_r$, or as a known parameter defined by a target accuracy level for the model fitting operation. The parameters of the model and σ (when assumed unknown) are selected to minimize the code length $L(D|\mathcal{H}^{(r)}; \boldsymbol{\Theta}^{(r)}, \sigma)$, which corresponds to the maximization of the log-likelihood function, and therefore, to the maximum-likelihood (ML) solution for the corresponding model fitting problem [37]. As a result, the model-

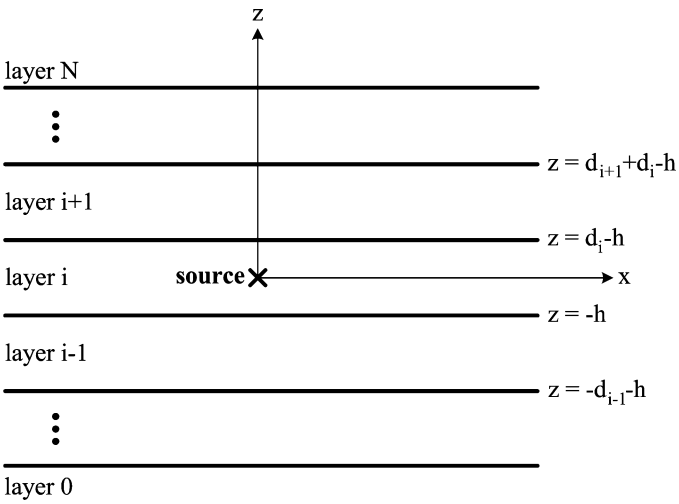

Fig. 1. General multilayer planar medium.

conditioned code length corresponding to the model $\mathcal{H}^{(r)}$ can be written as

$$\begin{aligned} L(D|\mathcal{H}^{(r)};\boldsymbol{\Theta}_{r,\mathrm{ML}},\sigma_{\mathrm{ML}}) &= \frac{N}{2}(\log(2\pi)+1) \\ &\quad + \frac{N}{2}\log\left(\frac{1}{N}\|\mathbf{y}-\mathbf{f}^{(r)}(\mathbf{x};\boldsymbol{\Theta}_{r,\mathrm{ML}})\|_2^2\right) \end{aligned} \tag{10}$$

when σ is treated as an unknown parameter, and

$$\begin{aligned} L(D|\mathcal{H}^{(r)};\boldsymbol{\Theta}_{r,\mathrm{ML}}) &= \frac{N}{2}\log(2\pi\sigma^2) \\ &\quad + \frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{f}^{(r)}(\mathbf{x};\boldsymbol{\Theta}_{r,\mathrm{ML}})\|_2^2 \end{aligned} \tag{11}$$

when σ is specified based on a target fitting performance level. Since the goal is to get the total description length in (6), once the $L(D|H)$ component has been obtained in (10) and (11) for two different scenarios, the remaining term, i.e., the code length for the model parameters of $\mathcal{H}^{(r)}$, is given by

$$L(\mathcal{H}^{(r)}) = \frac{\kappa(\mathcal{H}^{(r)})}{2}\log(N) \tag{12}$$

where $\kappa(\mathcal{H}^{(r)})$ is the total number of effective real parameters used in the model. The expression in (12) is based on the assumption that the model parameters are estimated using N data samples. Therefore, the error variance in each parameter is proportional to $1/N$, corresponding to the assumed estimation accuracy whose logarithm determines the required bit precision for the parameter.

III. DCIM With Model Order Selection

A. Brief Overview of DCIM

For the sake of illustration, consider a typical planar multilayered structure, as shown in Fig. 1, where the EM properties of the layers vary in the z-direction only.

Traditionally, Green's functions for planar layered media are obtained recursively as closed-form expressions in the spectral domain, and they are transformed to the spatial domain by the following Hankel transform integral (Sommerfeld integral):

$$g(\rho) = \frac{1}{2\pi}\int_S dk_\rho k_\rho J_0(k_\rho\rho)\tilde{G}(k_\rho) \tag{13}$$

where g and $\tilde{G}$ are the spatial- and spectral-domain Green's functions, respectively, J_0 is a zeroth-order Bessel function of the first kind, and S denotes the integration path defined over the first quadrant in the k_ρ-plane. Since it is generally computationally intensive to evaluate Sommerfeld integrals using numerical quadratures, the DCIM has been developed to avoid this expensive step by introducing two critical steps instead, which are, namely: 1) the approximation of the spectral-domain Green's functions by the linear combinations of the complex exponentials and 2) the evaluation of the resulting Sommerfeld integrals in closed forms using the Sommerfeld integral identity

$$\frac{e^{-jk_0 r}}{r} = \int_S dk_\rho k_\rho J_0(k_\rho\rho)\frac{e^{-jk_z z}}{jk_z} \tag{14}$$

where $r = \sqrt{\rho^2+z^2}$ and $k_0 = \sqrt{k_\rho^2+k_z^2}$. As a result, the spatial-domain Green's functions can be obtained analytically as a sum of complex images

$$\begin{aligned} \underbrace{\frac{1}{4\pi}\sum_{l=1}^{M}\alpha^{(l)}\frac{e^{-jk_0 r_i}}{r^{(l)}}}_{g(\rho)} &= \frac{1}{2\pi}\int_S dk_\rho k_\rho J_0(k_\rho\rho)\underbrace{\frac{1}{2jk_z}\sum_{l=1}^{M}\alpha^{(l)}e^{-\beta^{(l)}k_z}}_{\tilde{G}(k_\rho)} \end{aligned} \tag{15}$$

where $r^{(l)} = \sqrt{\rho^2+(-j\beta^{(l)})^2}$ is a complex distance, due to which the method is referred to as DCIM.

B. Automatic Order Selection for DCIM

According to the discussion in Section III-A, central to the DCIM is the approximation of the spectral-domain Greens functions by rational exponential mixtures of the form

$$\tilde{G}_a(k_\rho;\boldsymbol{\Theta}_r) = \sum_{l=1}^{r}\alpha^{(l)}\frac{e^{-\beta^{(l)}k_z}}{2jk_z} \tag{16}$$

where r is the order of the model corresponding to the number of exponentials used in the approximation, $\alpha^{(l)}$ and $\beta^{(l)}$ are the weights of the fractional terms and the constants of the exponents, respectively, and $\boldsymbol{\Theta}_r$ $(=[\alpha^{(1)}\ \ldots\ \alpha^{(r)}\ \beta^{(1)}\ \ldots\ \beta^{(r)}]^T)$ is the model parameter vector.

By drawing an analogy from the generic data model in (1), the model fitting problem for the DCIM can be written as

$$\tilde{G}(k_\rho^{(n)}) = \tilde{G}_a^{(r)}(k_\rho^{(n)};\boldsymbol{\Theta}_r) + w^{(n)}, \qquad n=1,\ldots,N. \tag{17}$$

where the fitting is performed for the spectral-domain points $\mathcal{K} = \{k_\rho^{(1)},\ldots,k_\rho^{(N)}\}$ defined over the integration path of the inverse Hankel transform operation in (13). Note that the minimization of the weighted 2-norm of the spatial-domain error of

the Green's functions, for which the cost function was defined as

$$J(e)=\int_0^\infty \omega(\rho)|e(\rho)|^2 d\rho \tag{18}$$

where $e(\rho)$ is the spatial error function, corresponding to the approximation procedure in DCIM, and $\omega(\rho)$ is the spatial weighting function, has recently been introduced as the optimization criterion for obtaining the model parameter vector Θ_r in [29]. In the same reference, it has also been demonstrated that this spatial-domain goal can be written as a spectral-domain optimization problem as

$$\underset{\mathbf{\Theta}_r\in\mathcal{C}^m}{\text{minimize}} \quad \|\tilde{\mathbf{G}}(\mathbf{k}_\rho)-\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_r)\|_{\mathbf{\Gamma}_\mathcal{K}} \tag{19}$$

where $\mathbf{\Gamma}_\mathcal{K}\in\mathcal{C}^{N\times N}$ is the positive definite weighting matrix, being a function of the spectral-domain samples $\mathcal{K}$, $\mathbf{k}_\rho$ is the vector containing the elements in $\mathcal{K}$, $\tilde{\mathbf{G}}(\mathbf{k}_\rho)$ is the vector of the samples of the spectral-domain Greens functions, and $\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_r)$ is the vector containing the samples of the approximation model (of order r) corresponding to the model parameter vector $\mathbf{\Theta}_r$. The optimization problem in (19) targets the spectral shaping of the fitting errors such that the spatial-domain weighted error cost in (18) is minimized. Therefore, this procedure would be equivalent to the model fitting problem in (17) where the spectral model fitting noise vector

$$\mathbf{w}=[\,w^{(1)}\quad w^{(2)}\quad \ldots\quad w^{(N)}\,]^T \tag{20}$$

has the covariance $\sigma^2\mathbf{\Gamma}_\mathcal{K}^{-1}$, which corresponds to the independent fitting noise in the spatial domain following the discussion in [29]. Here, the σ target is a reflector of the fitting accuracy. The code length expressions for the data-conditioned model of order r in (10) and (11) should be accordingly modified as

$$\begin{aligned}L(D|\mathcal{H}^{(r)};\mathbf{\Theta}_{r,\mathrm{ML}},\sigma_{\mathrm{ML}})&=\frac{N}{2}\left(\log(2\pi\det(\mathbf{\Gamma}_\mathcal{K}^{-1}))+1\right)\\&\quad+\frac{N}{2}\log\left(\frac{1}{N}\|\tilde{\mathbf{G}}(\mathbf{k}_\rho)-\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_{r,\mathrm{ML}})\|^2_{\mathbf{\Gamma}_\mathcal{K}}\right)\end{aligned} \tag{21}$$

when σ is treated as an unknown parameter, and

$$\begin{aligned}L(D|\mathcal{H}^{(r)};\mathbf{\Theta}_{r,\mathrm{ML}})&=\frac{N}{2}\log(2\pi\sigma^2\det(\mathbf{\Gamma}_\mathcal{K}^{-1}))\\&\quad+\frac{1}{2\sigma^2}\|\tilde{\mathbf{G}}(\mathbf{k}_\rho)-\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_{r,\mathrm{ML}})\|^2_{\mathbf{\Gamma}_\mathcal{K}}\end{aligned} \tag{22}$$

when σ is specified. Note that the code length expressions given above assume that the $\mathbf{\Theta}_{r,\mathrm{ML}}$ vector is obtained through the nonlinear least squares procedure corresponding to the ML criterion, which is, in effect, equivalent to the optimization problem in (19). However, in the practical applications of the DCIM, a two-step approach is employed with the following brief descriptions of the steps: 1) a subspace procedure such as the generalized pencil of function (GPOF) [38] or ESPRIT [39] is used to obtain the constants of the exponents i.e., $\beta^{(l)}$'s and 2) the cost function in (19) is minimized with respect to the weighting parameters $\alpha^{(l)}$'s. This procedure typically achieves close approximation to the ML solution with more reasonable computational complexity, resulting in a parameter vector $\mathbf{\Theta}_{r,\mathrm{DCIM}}$ to be used in evaluating the code lengths in (21) and (22).

For the code length computation of the model parameters for $\mathcal{H}^{(r)}$, one needs to mainly consider two different scenarios: a single-level DCIM and multilevel DCIM. According to the single-level DCIM, since the same exponents and weighting parameters are used for the entire integration path, it results in $2r$ complex constants based on N sample points, leading to

$$L(\mathcal{H}^{(r)})=2r\log(N) \tag{23}$$

according to (12). However, in the multilevel DCIM, the integration path is divided into L separate regions, for which there are separate numbers of samples N_k, separate orders r_k, and separate parameter vectors $\mathbf{\Theta}_{r_k}$ for $k=1,\ldots,L$. Therefore, the model hypothesis in this case will be of the compound form, represented by $\mathcal{H}^{(r_1,r_2,\ldots,r_L)}$. Although the exponents of a region are computed from the samples of that region alone, the weighting coefficients of all rational exponential mixtures are computed by a final weighted least squares performed over all the samples of the combined regions, as proposed in [29], to avoid the leakage caused by the independent weight optimizations. As a result, the code length for the hypothesis $\mathcal{H}^{(r_1,r_2,\ldots,r_L)}$ can be written as

$$L(\mathcal{H}^{(r_1,r_2,\ldots,r_L)})=\underbrace{\sum_{k=1}^{L} r_k\log(N_k)}_{\text{for exponents}}+\underbrace{r\log(N)}_{\text{for weights}} \tag{24}$$

where $r=\sum_{k=1}^{L} r_k$ is the total order and $N=\sum_{k=1}^{L} N_k$ is the total number of samples on the integration path. Consequently, for a multilevel DCIM with L levels, the total code length for the hypothesis $\mathcal{H}^{(r_1,r_2,\ldots,r_L)}$ can be written as

$$\begin{aligned}L(D)^{(r)}&=\frac{N}{2}\left(\log(2\pi\det(\mathbf{\Gamma}_\mathcal{K}^{-1}))+1\right)\\&\quad+\frac{N}{2}\log\left(\frac{1}{N}\|\tilde{\mathbf{G}}(\mathbf{k}_\rho)-\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_{r,\mathrm{DCIM}})\|^2_{\mathbf{\Gamma}_\mathcal{K}}\right)\\&\quad+\sum_{k=1}^{L} r_k\log(N_k)+r\log(N)\end{aligned} \tag{25}$$

when σ is treated as an unknown parameter, and

$$\begin{aligned}L(D)^{(r)}&=\frac{N}{2}\log(2\pi\sigma^2\det(\mathbf{\Gamma}_\mathcal{K}^{-1}))\\&\quad+\frac{1}{2\sigma^2}\|\tilde{\mathbf{G}}(\mathbf{k}_\rho)-\tilde{\mathbf{G}}_a^{(r)}(\mathbf{k}_\rho;\mathbf{\Theta}_{r,\mathrm{DCIM}})\|^2_{\mathbf{\Gamma}_\mathcal{K}}\\&\quad+\sum_{k=1}^{L} r_k\log(N_k)+r\log(N)\end{aligned} \tag{26}$$

when σ is specified *a priori*. Therefore, the DCIM with model order selection involves choosing the model with the least code length, where the code length computation is based on the expressions in (25) and (26).

The treatment of σ as a known or unknown parameter deserves a separate discussion. According to the proposed procedure, if σ is chosen as an unknown parameter, the DCIM with automatic order selection will return a prescribed σ value, in

addition to the regional orders and parameters. This σ value would reflect the optimal balance point between the fitting error level versus model complexity according to the two-level MDL criterion. This choice may result in over-modeling with a fitting precision better than needed for the specific application. In order to reduce the implementation complexity of any EM simulation method that employs the closed-form Green's functions obtained by the proposed DCIM, one may specify a σ value, instead of treating it as an unknown. The user-specified σ value should depend on the accuracy and complexity requirements of the application. One can define a weighted signal-to-noise ratio (SNR)

$$\mathrm{SNR} = \frac{\frac{1}{N}\|\tilde{\mathbf{G}}(k_\rho)\|^2_{\tilde{\mathbf{\Gamma}}_\mathcal{K}}}{\frac{1}{N}\|\tilde{\mathbf{G}}(k_\rho) - \tilde{\mathbf{G}}_\mathbf{a}(k_\rho)\|^2_{\tilde{\mathbf{\Gamma}}_\mathcal{K}}} \tag{27}$$

as an accuracy measure, which is the ratio of average weighted energy of the spectral-domain Green's function and fitting error. Therefore, for a chosen weighted SNR level, the target σ parameter can be computed as

$$\sigma^2_{\mathrm{target}} = \frac{\frac{1}{N}\|\tilde{\mathbf{G}}(k_\rho)\|^2_{\tilde{\mathbf{\Gamma}}_\mathcal{K}}}{\mathrm{SNR}_{\mathrm{target}}} \tag{28}$$

where $\mathrm{SNR}_{\mathrm{target}}$ is an application-dependent parameter.

IV. Results and Discussions

Once the theory and its possible scenarios have been thoroughly developed and studied in Section III, its implementation and performance are provided in this section on some typical examples, chosen from rather difficult cases for the traditional DCIM. Before getting into the details of the results and discussions, it should be stated that the performance of the automatic order selection algorithm will be presented on the latest version of the DCIM only, namely, the three-level DCIM with spatial error criterion [22], [29]. Although the algorithm has been tested on many different multilayered planar geometries with success, for the sake of brevity, only two geometries, which were considered as the most difficult ones for the traditional two-level DCIM [28], are presented in this paper. It should also be noted that the proposed algorithm has been tested on the vector potentials (g^A) before and after extracting the surface wave poles (SWPs), as the existence of SWPs makes the fitting difficult, while the Green's functions for scalar potentials (g^q) have been obtained only after extracting the SWPs, as SWPs often dominate the scalar potentials to a level that the DCIM cannot capture the correct nature of the function, as discussed in [22].

To set the stage for the implementation of the proposed algorithm for the three-level DCIM, note that the integration path is divided into three separate regions with three separate orders r_k for $k = 1, 2, 3$, and that the total code length of the hypothesis $\mathcal{H}^{(r_1,r_2,r_3)}$ is calculated by (25) and (26) with $L = 3$. Since the goal is to find the ternary order $\mathbf{r} = (r_1, r_2, r_3)$ that achieves the least code length, the code length computation is performed for all $r_k = 1, \ldots, M_k$ and $k = 1, 2, 3$, and the minimum one among them is considered to be the MDL. Although the values of M_k for the three levels can be significantly narrowed down by educated guesses, they are chosen to be as large as possible in this study for the purpose of demonstration. Moreover, the proposed algorithm is implemented for both cases where σ is treated as an unknown and known parameter, using the equations in (25) and (26), respectively. In the case of unknown σ, the MDL algorithm returns the ternary order $\mathbf{r}_{\mathrm{MDL}}$, together with a value for σ as

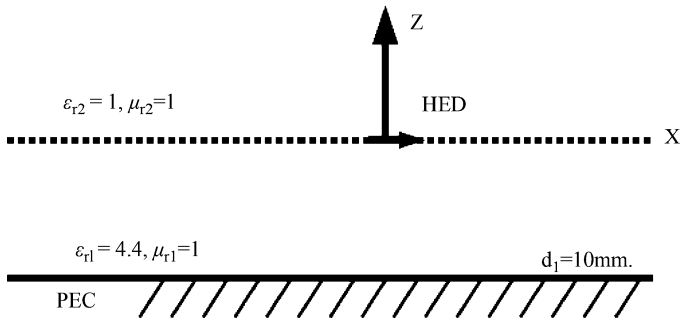

Fig. 2. Geometry for Example-1 and Example-2. A perfect electric conductor (PEC) backed lossless dielectric material in free space.

$$\sigma_{\mathrm{MDL}} = \left[\frac{1}{N}\|\tilde{\mathbf{G}}(\mathbf{k}_\rho) - \tilde{\mathbf{G}}_a(\mathbf{k}_\rho; \mathbf{\Theta}_{\mathbf{r}_{\mathrm{MDL}},\mathrm{DCIM}})\|^2_{\tilde{\mathbf{\Gamma}}_\mathcal{K}}\right]^{1/2} \tag{29}$$

while, in the case of predefined σ_{PRE}, the ternary order that gives the least code length is referred to as $\mathbf{r}_{\mathrm{PRE}}$ in this study. In addition, as briefly reviewed in Section I, the performance of the traditional approach in determining the order of the model in DCIM, referred to as $\mathbf{r}_{\mathrm{TRA}}$ in this paper, is also compared to the proposed algorithm.

Before proceeding with the details of the examples, it would be instructive to provide a brief summary of what has been done and how parameters have been named in the tables and figures in this paper:

1) three-level DCIM with automatic order selection is tested on g^A with and without SWP extraction, referred to as "g^A without SWP" and "g^A with SWP," respectively;
2) three-level DCIM with automatic order selection is tested on g^q with SWP extraction, referred to as "g^q without SWP";
3) $\mathbf{r}_{\mathrm{MDL}}$ and $\mathbf{r}_{\mathrm{PRE}}$ are the ternary orders achieving the minimum code length obtained from (25) and (26), respectively;
4) $\mathbf{r}_{\mathrm{TRA}}$ is the ternary order found from the traditional approach.

As the first example, consider the geometry in Fig. 2 for which the Green's functions of the vector and scalar potentials, g^A_{xx} and g^q_x, respectively, are obtained at $f = 4.075$ GHz using the three-level DCIM with spatial error criterion. For this setting, there is a TE-mode SWP with a rather small residue, resulting in a noticeable contribution only after very large distances, i.e., for large values of ρ. Therefore, the closed-form approximation of g^A_{xx} can be obtained from the proposed algorithm with and without extracting the SWP. However, since the TM-mode SWP contribution is quite dominant even in the intermediate distance from the source, a good fit for g^q_x can only be achieved after extracting the SWPs. Although it is suggested, as a general rule of thumb, that it is always a good practice to extract the SWPs from the spectral-domain Green's functions before the application of the DCIM [22], the vector-potential

TABLE I
ORDER SELECTION FOR THE THREE-LEVEL DCIM FOR THE GEOMETRY IN FIG. 2. $f = 4.075$ GHz, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$

Example-1		g^A_{xx}		g^q_x
		w/ SWP	w/o SWP	w/o SWP
MDL	$\mathbf{r}_{MDL}$	3,22,15	10,20,16	9,13,17
	$\mathbf{r}_{PRE}$	1,2,4	3,9,11	1,1,2
$\mathbf{r}_{TRA}$		16,16,19	15,16,18	15,6,18

Green's function in this example has been treated both ways, just to demonstrate the performance of the order selection algorithm under rather difficult conditions. As a result, the performances of the order selections for the three-level DCIM by the two MDL approaches, (25) and (26), and by the traditional approach for g^A_{xx} and g^q_x have been presented in Table I. As observed from this example and many others, the MDL approach with unknown σ ($\sigma_{\rm MDL}$) results in less or equal number of exponentials, in total, as compared to the number obtained from the traditional selection. It is fair to state that, even before *a priori* target accuracy level of the approximation is defined by the user, the performance of the MDL approach is quite satisfactory and better than the traditional approach. Moreover, if one accepts, for a specific problem at hand, to work with some degree of inaccuracies in the end result by setting $\sigma_{\rm PRE}$ to $10^n\sigma_{\rm MDL}$, the minimum code length is reduced drastically, resulting in significantly less number of exponentials, as expected. Of course, the choice of $\sigma_{\rm PRE}$ must be based on the tradeoff between acceptable error for the problem and the computational cost of the algorithm. To demonstrate the robustness of the algorithm, the corresponding Green's functions obtained by the orders $\mathbf{r}_{\rm PRE}$ are presented in Fig. 3, and compared to those obtained by the numerical integration of the transformation. It should be noted that even though the number of exponentials employed in the approximation has been significantly less than that of the ideal case, i.e., $\mathbf{r}_{\rm MDL}$, the approximation has been quite successful, as shown in Fig. 3.

As a second example, the same geometry in Fig. 2 has been employed to study the algorithm at $f = 3.0$ GHz for which there is a dominant SWP for the TM modes, while there is no SWP for TE modes, resulting in a dominant SWP contribution in the far field of g^q_x and a dominant branch point contribution in the far field of g^A_{xx}. As a result of this study, the order selections of the two MDL approaches and the traditional method have been provided in Table II for both g^A_{xx} and g^q_x with almost the same conclusion: the MDL algorithm with unknown σ $(=\ \sigma_{\rm MDL})$ recommends a smaller number of exponentials in total as compared to the one from the traditional selection, and when the intended accuracy is sacrificed by choosing $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$, the order $\mathbf{r}_{\rm PRE}$ is reduced significantly. To assess and to demonstrate the amount of error in the end result, i.e., in the spatial-domain Green's functions, introduced by using the order $\mathbf{r}_{\rm PRE}$ (relaxing the fitting error parameter σ as compared to $\sigma_{\rm MDL}$), the plots of the spatial-domain Green's functions obtained by the DCIM are provided for this choice with no noticeable difference, as shown in Fig. 4.

As the final example, the geometry shown in Fig. 5 is studied at $f = 4.0$ GHz for which the SWP contribution is quite significant in the near field for both g^A_{xx} and g^q_x, even though the

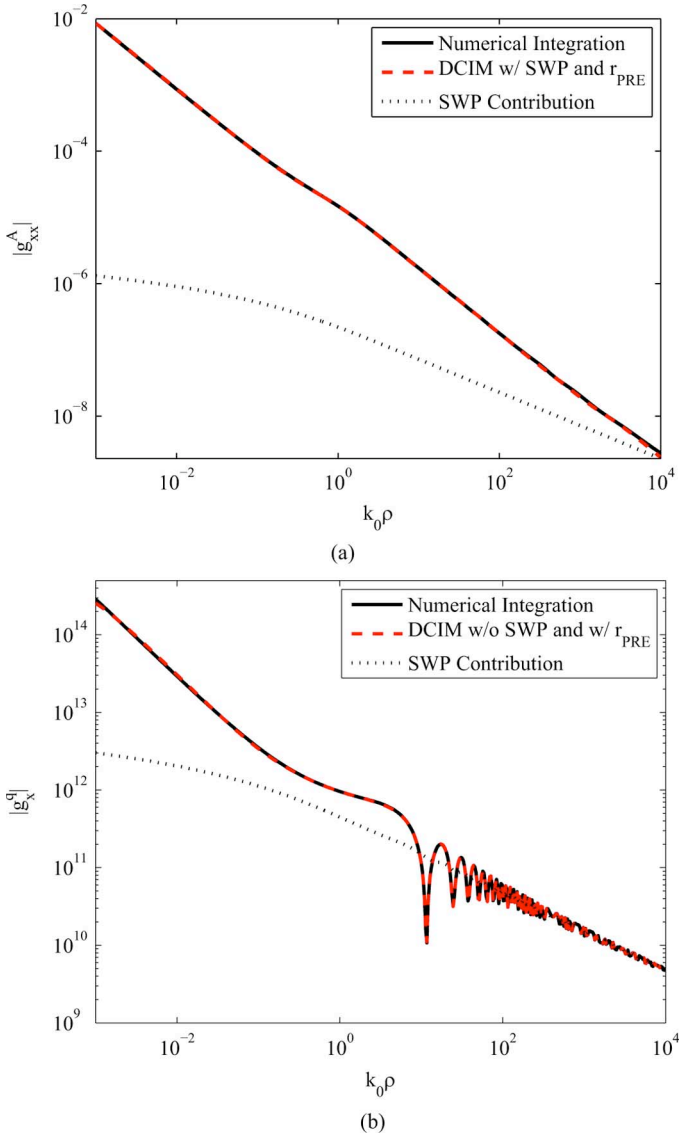

Fig. 3. Magnitudes of the Green's functions of: (a) the vectoral potential for $\mathbf{r}_{\rm PRE} = (1, 2, 4)$ and (b) the scalar potential for $\mathbf{r}_{\rm PRE} = (1, 1, 2)$ for the geometry in Fig. 2. $f = 4.075$ GHz, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$.

TABLE II
ORDER SELECTION FOR THE THREE-LEVEL DCIM FOR THE GEOMETRY IN FIG. 2. $f = 3$ GHz, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$

Example-2		g^A_{xx}	g^q_x
		no SWP	w/o SWP
MDL	$\mathbf{r}_{MDL}$	12,3,11	13,3,7
	$\mathbf{r}_{PRE}$	9,2,6	6,3,1
$\mathbf{r}_{TRA}$		16,10,8	13,3,12

SWP contribution decays exponentially and the branch-point contribution determines the behavior of the Green's functions for $\rho \longrightarrow \infty$ in this example. Therefore, it is more difficult to capture the behavior of the Green's functions unless the SWPs are extracted, contrary to the cases studied in the previous examples. To study the influence of the SWPs on the proposed algorithm, the order selection has been performed for g^A_{xx} without extracting the SWPs, which inevitably increases the order selected by all the variants of the algorithm (Table III). Note that while $\sigma_{\rm PRE}$ is chosen as $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$ when the SWPs

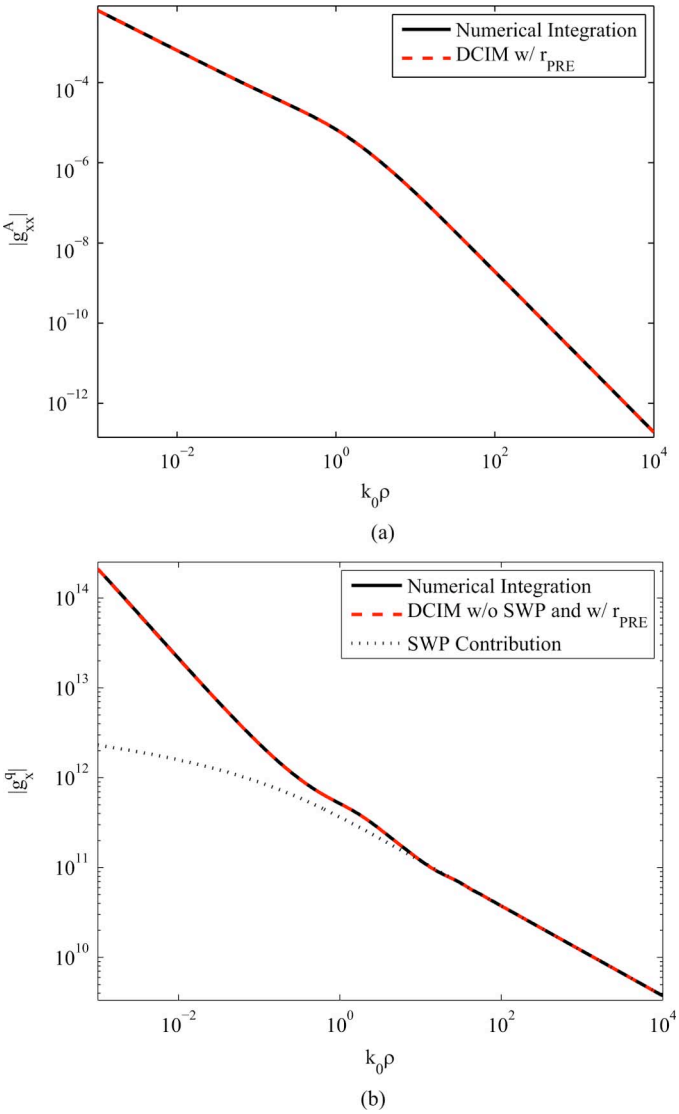

Fig. 4. Magnitude of the Green's functions of: (a) the vectoral potential for $\mathbf{r}_{\rm PRE} = (9, 2, 6)$ and (b) the scalar potential for $\mathbf{r}_{\rm PRE} = (6, 3, 1)$ for the geometry in Fig. 2. $f = 3.0$ GHz, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$.

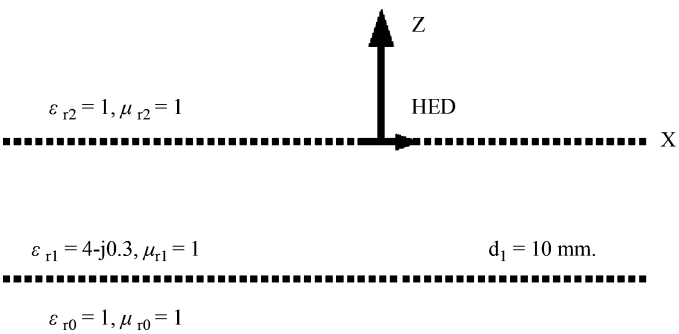

Fig. 5. Geometry for Example-3. A lossy material in air.

TABLE III
ORDER SELECTION FOR THE THREE-LEVEL DCIM FOR THE GEOMETRY IN FIG. 5. $f = 4$ GHz, $\sigma_{\rm PRE} = \sqrt{10}\sigma_{\rm MDL}$ WHEN SWPS ARE NOT EXTRACTED, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$ WHEN SWPS ARE EXTRACTED

Example-3		g^A_{xx}		g^q_x
		w/ SWP	w/o SWP	w/o SWP
MDL	$\mathbf{r}_{MDL}$	7,38,16	18,3,14	18,10,16
	$\mathbf{r}_{PRE}$	5,34,12	7,5,10	8,2,7
$\mathbf{r}_{TRA}$		14,27,15	15,4,9	15,5,13

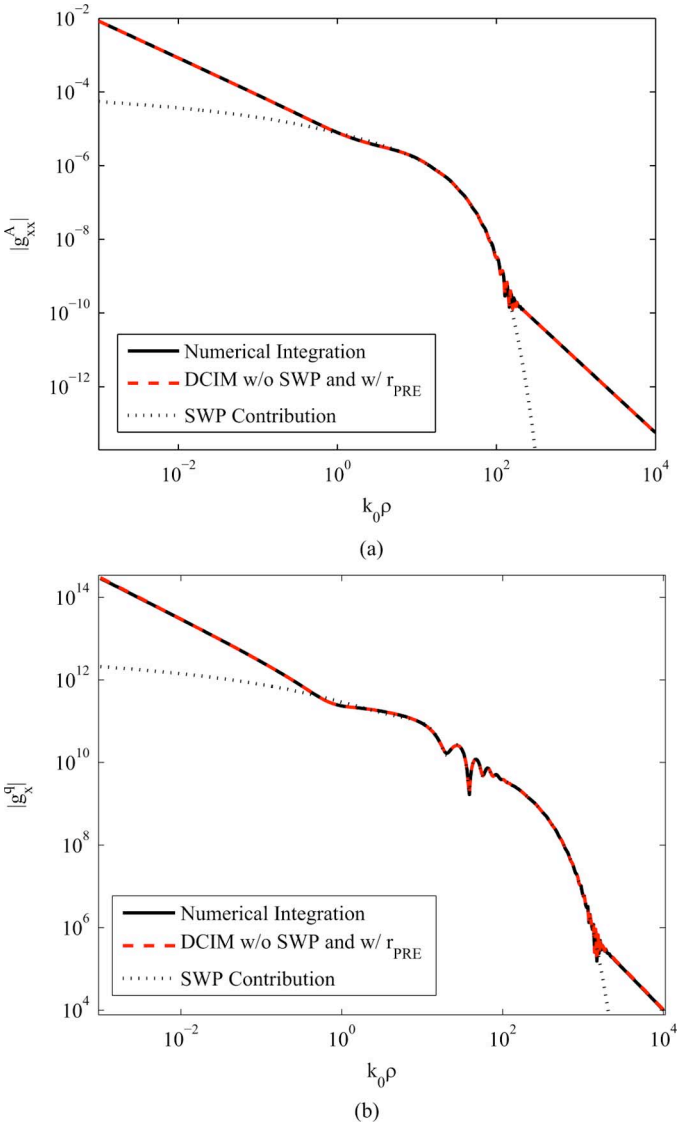

Fig. 6. Magnitude of the Green's functions of: (a) the vector potential for $\mathbf{r}_{\rm PRE} = (7, 5, 10)$ and (b) the scalar potential for $\mathbf{r}_{\rm PRE} = (8, 2, 7)$ for the geometry in Fig. 5. $f = 4.0$ GHz, $\sigma_{\rm PRE} = 10^3\sigma_{\rm MDL}$.

are extracted, it would be reasonable to choose the fitting error parameter σ closer to the ideal case $\sigma_{\rm MDL}$ when the SWPs are not extracted, by setting $\sigma_{\rm PRE} = \sqrt{10}\sigma_{\rm MDL}$, because of the nature of the waves supported by the geometry in such cases. As observed in Table III, the MDL algorithm with $\sigma_{\rm MDL}$ suggests slightly more number of exponentials in total compared to the traditional approach, while the MDL algorithm with $\sigma_{\rm PRE}$ results in significantly reduced order without causing any noticeable deterioration in the spatial-domain Green functions, as shown in Fig. 6.

For the sake of completeness, the additional error introduced by relaxing the fitting error parameter σ is studied for all the Green's functions and the results for g^A_{xx} with the SWP extracted in example-3 have been provided for three different selections of $\sigma_{\rm PRE}$ in Fig. 7. In this study, the error is defined by the absolute

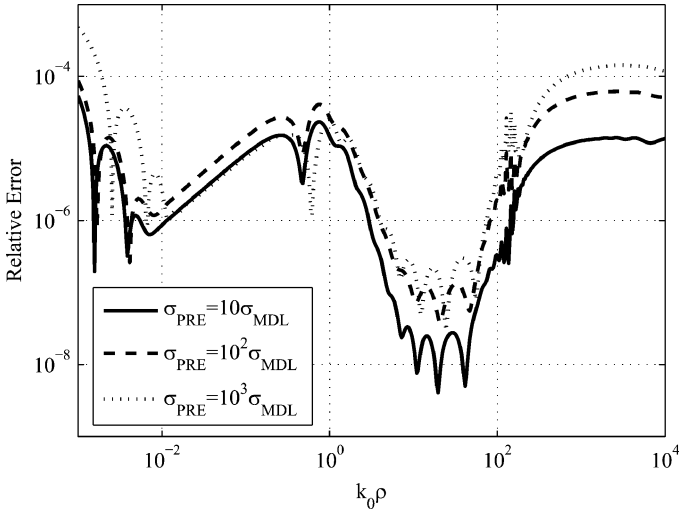

Fig. 7. Relative error in g_{xx}^{A} (example-3) introduced by the choice of different error parameters σ as compared to the parameter provided by the algorithm itself. $\mathbf{r}_{\mathrm{MDL}} = (18, 3, 14)$, and $\mathbf{r}_{\mathrm{PRE}} = (12, 5, 7)$ for $\sigma_{\mathrm{PRE}} = 10\sigma_{\mathrm{MDL}}$, $\mathbf{r}_{\mathrm{PRE}} = (11, 3, 8)$ for $\sigma_{\mathrm{PRE}} = 10^2\sigma_{\mathrm{MDL}}$, and $\mathbf{r}_{\mathrm{PRE}} = (7, 5, 10)$ for $\sigma_{\mathrm{PRE}} = 10^3\sigma_{\mathrm{MDL}}$.

value of the difference between the spatial-domain Green'function for the orders of $\mathbf{r}_{\mathrm{PRE}}$ and $\mathbf{r}_{\mathrm{MDL}}$ as normalized to that for $\mathbf{r}_{\mathrm{MDL}}$, i.e., $|g(\rho, \sigma_{\mathrm{PRE}}) - g(\rho, \sigma_{\mathrm{MDL}})|/|g(\rho, \sigma_{\mathrm{MDL}})|$. Note that, for this specific example, the following orders corresponding to the different fitting error parameters σ have been provided by the MDL algorithm: $\mathbf{r}_{\mathrm{MDL}} = (18, 3, 14)$, and $\mathbf{r}_{\mathrm{PRE}} = (12, 5, 7)$ for $\sigma_{\mathrm{PRE}} = 10\sigma_{\mathrm{MDL}}$, $\mathbf{r}_{\mathrm{PRE}} = (11, 3, 8)$ for $\sigma_{\mathrm{PRE}} = 10^2\sigma_{\mathrm{MDL}}$, and $\mathbf{r}_{\mathrm{PRE}} = (7, 5, 10)$ for $\sigma_{\mathrm{PRE}} = 10^3\sigma_{\mathrm{MDL}}$. From this study, it is observed that once the SWPs have been extracted, the choice of the fitting error parameter σ plays a significant role in order selection while introducing a negligible error in the end result. For all the cases studied thus far, provided SWPs are extracted *a priori*, the choice of $\sigma_{\mathrm{PRE}} = 10^3\sigma_{\mathrm{MDL}}$ works quite well in optimizing accuracy and the description length. It would be worth mentioning that although the total numbers of exponentials corresponding to $\sigma_{\mathrm{PRE}} = 10^2\sigma_{\mathrm{MDL}}$ and $\sigma_{\mathrm{PRE}} = 10^3\sigma_{\mathrm{MDL}}$ are the same, their distributions over the three regions of the three-level DCIM differ, providing more emphasis to the regions where the amplitudes are relatively large, as σ gets large.

After providing the performance of the algorithm on some typical examples, it is worth to note that, in this study, the error relaxation is performed by using the orders of magnitudes of σ_{MDL}, as σ_{MDL} has already been obtained in (29). Alternatively, the user can set an SNR level depending on the application as discussed in Section III.

V. Conclusion

After having introduced the three-level DCIM that can take different wave natures into account and that can provide an error metric to ensure the accuracy of the end result, the only remaining hurdle was the manual selection of the order of the approximation. In this paper, an order selection algorithm based on the MDL has been introduced and demonstrated on some typical examples that it can easily be implemented with success. Based on the cases studied during this study, it would be safe to state that the DCIM with automatic order selection usually suggests less or equal number of exponentials than the ones decided manually in the traditional approach. More importantly, since the MDL approach, exploiting the regularities in the data, selects the order of the model based on a tradeoff between the quality of fitting and complexity of the model, it guarantees the accuracy of the fitting as contrary to the traditional approach. Furthermore, this approach allows the user to modify the tradeoff between the accuracy and complexity in favor of one of the two, depending on the requirements of the actual problem at hand. It is inevitable that such a flexibility comes with a price tag; it requires some sort of input from the user, perhaps in the form of the SNR according to which a target fitting error parameter σ can be computed. However, it was found out that while the choice of the parameter had a significant influence on the order of the model, it is not critical for the accuracy of the end result, provided the SWPs have been extracted in advance. The method with all its variants was successfully applied to many different geometries and different components of the Green's functions.

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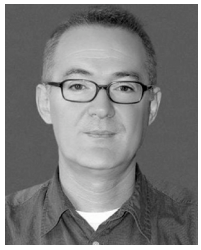

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