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ADAPTIVE ALGORITHM FOR THE BLIND SEPARATION OF SOURCES WITH FINITE SUPPORT

Alper T. Erdogan

Electrical Engineering, Koc University
Istanbul, Turkey
phone: + (90) 212 3381490, fax: + (90) 2123381548, email: alperdogan@ku.edu.tr
web: aspc.ku.edu.tr

ABSTRACT

A new blind separation algorithm which exploits only the finite support property and the independence of the sources is proposed. The approach is based on the minimization of magnitude peaks of some affine mapping outputs. It is proven that the perfect separators are the only global minima of the corresponding cost function. The proposed method leads to the construction of a simple adaptive algorithm for the separation of sources. A simulation example is provided to illustrate the algorithm's performance.

1. INTRODUCTION

In the area of Blind Source Separation (BSS), the exploitation of the structural and statistical signal features plays a critical role in devising methods to obtain adaptive separators. For example, kurtosis and negentropy based methods exploit the presumed non-Gaussianity of the sources (see for example [1, 2]).

Recently, some alternative approaches exploiting magnitude or domain boundedness of source signals (in addition to their mutual independence) have been proposed. The assumption about boundedness is a very simple and a reasonable one in real life scenarios.

The first BSS approach based on the boundedness of the input sources is in [3]. In this work, Pham reformulated the mutual information cost function in terms of quantile functions which enables the approximation of the cost function using order statistics. In the bounded domain case, this approximate cost function can be simplified to (the logarithm of) the difference between maximum and the minimum values of the separator output. In [3], an adaptive algorithm for simultaneous recovery of sources corresponding to the proposed cost function is also provided.

In [4], Vrins et.al. proposed the minimization of the effective support length for the separation of bounded magnitude sources. Around the same time frame, in [5] and [6], the minimization of the infinity norm of the separator output is proposed where it's proven that all the global minima for the cost function are the desired separators. The proposed criterion fits especially to QAM communication signals with positive-negative magnitude peak symmetry. The corresponding algorithm illustrated to achieve high Signal to Distortion Ratio (SDR) levels for relatively short window sizes for the separation of digital communications signals.

In this article, a new approach exploiting the finite support property of sources is introduced. This new approach makes use of an affine form to convert support length minimization problem of [3] into a (positive-negative symmetric) magnitude minimization problem similar to [5].

The organization of the article is as follows: BSS setup and the corresponding notation is introduced in Section 2. The new approach is provided in Section 3 where its global optimality property is also proven. In Section 4, an adaptive algorithm corresponding to the proposed criterion is provided, and its performance is illustrated through a simulation example in Section 5. Finally, conclusion is in Section 6.

2. BLIND SOURCE SEPARATION SETUP

We assume that there are p real sources with finite support, where

$$\max s_k = U \text{ and } \min s_k = -L, \quad U, L \geq 0, \tag{1}$$

where the minimum and maximum are over the set of values that each source can take. We further assume that sources are zero mean and unit variance without any loss of generality. The sources are mixed through a memoryless system with transfer matrix $\mathbf{C} \in \Re^{N \times p}$ (i.e., instantaneous mixtures), and therefore, we can write the mixtures as

$$\underbrace{\begin{bmatrix} y_1(k) \\ \vdots \\ y_N(k) \end{bmatrix}}_{\mathbf{y}(k)} = \mathbf{C} \underbrace{\begin{bmatrix} s_1(k) \\ \vdots \\ s_p(k) \end{bmatrix}}_{\mathbf{s}(k)} \quad \forall k. \tag{2}$$

In [3], it is shown that, for sources with finite support, the mutual information minimization criterion can be approximated with the minimization of the cost function

$$f(\mathbf{W}) = -\log(|\det(\mathbf{W}\mathbf{R}_{\mathbf{y}}\mathbf{W}^T)|) + \sum_{m=1}^{p} \log(\mathscr{R}(z_m)) \tag{3}$$

where $\mathbf{R}_{\mathbf{y}}$ is the covariance matrix of the mixtures, $\mathbf{W}^{p \times N}$ is the separator matrix, $\mathbf{z}(k) = \mathbf{W}\mathbf{y}(k)$ is the separator output sequence and $\mathscr{R}(z_m)$ is the support length of the m^{th} component of the separator outputs.

3. NEW BLIND SEPARATION APPROACH

In our approach, we assume a decomposition in the form $\mathbf{W} = \mathbf{\Theta}\mathbf{W}_{pre}$ where $\mathbf{W}_{pre}$ is the $p \times N$ prewhitening matrix such that $\mathbf{x}(k) = \mathbf{W}_{pre}\mathbf{y}(k)$ are whitened mixtures, and $\mathbf{\Theta}$ is an orthogonal matrix. For the purposes of obtaining a

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practical algorithm for support length minimization and relating to the (positive-negative symmetric) magnitude maximization in [5] to the support length minimization of [3] outlined above, we propose the following cost function:

$$L(\boldsymbol{\Theta},\mathbf{b}) \quad = \quad \sum_{m=1}^{p} \|\boldsymbol{\Theta}_{m,:}\mathbf{x}+b_m\|_\infty \tag{4}$$

where infinity norm refers to the maximum value within the time window of available data (for the rest of the article, we assume that the data window is sufficiently rich to reflect the ensemble behavior in terms of peak values). Here $\mathbf{b}$ is a slack vector of the optimization process introduced for positive-negative peak balancing purposes, where the real goal of the optimization is to obtain $\boldsymbol{\Theta}$ parameter, and the separator outputs $\mathbf{z}(k)=\boldsymbol{\Theta}\mathbf{x}(k)$.

Defining

$$\mathbf{G}=\boldsymbol{\Theta}\mathbf{W}_{pre}\mathbf{C} \tag{5}$$

as the effective orthonormal mapping between the sources and the separator outputs, we can rewrite the proposed cost function as

$$J(\mathbf{G},\mathbf{b}) \quad = \quad \sum_{m=1}^{p} \|\mathbf{G}_{m,:}\mathbf{s}(k)+b_m\|_\infty \tag{6}$$

$$= \quad \sum_{m=1}^{p} J_m(\mathbf{G}_{m,:},b_m), \tag{7}$$

and pose the optimization problem

$$\underset{\mathbf{G},\mathbf{b}}{\text{minimize}}\ J(\mathbf{G},\mathbf{b}) \tag{8}$$

$$\text{subject to } \mathbf{G}\mathbf{G}^T=I. \tag{9}$$

We now show that the global minima of this optimization problem are $\mathbf{G}$ matrices corresponding to the perfect separation condition:

We first note that for a given $\mathbf{G}$ and $\mathbf{b}$, $\mathbf{G}_{m,:}$ can be decomposed as

$$\mathbf{G}_{m,:}=\mathbf{G}^{(p)}_{m,:}+\mathbf{G}^{(n)}_{m,:} \tag{10}$$

where

$$\mathbf{G}^{(p)}_{m,k}=\left\{\begin{array}{cc} \mathbf{G}_{m,k} & \text{if } \mathbf{G}_{m,k}>0 \\ 0 & \text{otherwise}\end{array}\right. , \quad m,k=1,\ldots,p, \tag{11}$$

and similarly,

$$\mathbf{G}^{(n)}_{m,k}=\left\{\begin{array}{cc} \mathbf{G}_{m,k} & \text{for } \mathbf{G}_{m,k}<0 \\ 0 & \text{otherwise}\end{array}\right. \quad m,k=1,\ldots,p. \tag{12}$$

If we also define the output of the affine form as

$$\mathbf{o}(k)=\mathbf{G}\mathbf{s}(k)+\mathbf{b}, \tag{13}$$

the maximum value of the m^{th} component of $\mathbf{o}$ is given by

$$o_m^{\max}=U\|\mathbf{G}^{(p)}_{m,:}\|_1+L\|\mathbf{G}^{(n)}_{m,:}\|_1+b_m, \tag{14}$$

which is obtained by the input

$$\bar{\mathbf{s}}^{(m)}=U sign(\mathbf{G}^{(p)}_{m,:})+L sign(\mathbf{G}^{(n)}_{m,:}), \tag{15}$$

and the minimum of the m^{th} component of $\mathbf{o}$ is given by

$$o_m^{\min}=-U\|\mathbf{G}^{(n)}_{m,:}\|_1-L\|\mathbf{G}^{(p)}_{m,:}\|_1+b_m, \tag{16}$$

which is obtained by the input

$$\underline{\mathbf{s}}^{(m)}=-L sign(\mathbf{G}^{(p)}_{m,:})-U sign(\mathbf{G}^{(n)}_{m,:}). \tag{17}$$

As a result, the function $\mathbf{J}_m$ has the value

$$\begin{aligned}\mathbf{J}_m(\mathbf{G},\mathbf{b})= \quad & \max\{|U\|\mathbf{G}^{(p)}_{m,:}\|_1+L\|\mathbf{G}^{(n)}_{m,:}\|_1+b_m|,\\ & |L\|\mathbf{G}^{(p)}_{m,:}\|_1+U\|\mathbf{G}^{(n)}_{m,:}\|_1-b_m|\}. \end{aligned}\tag{18}$$

If a point $\left[\begin{array}{cc}\mathbf{G}^* & \mathbf{b}^*\end{array}\right]$ is a local minimum of the proposed optimization problem, then there exists a subgradient $\mathbf{g}\in\partial J(\mathbf{G}^*,\mathbf{b}^*)$, where $\partial J(\mathbf{G}^*,\mathbf{b}^*)$ is the subdifferential set of J at the point $\left[\begin{array}{cc}\mathbf{G}^* & \mathbf{b}^*\end{array}\right]$, which is orthogonal to the tangent cone (space) of the constraint set at $\left[\begin{array}{cc}\mathbf{G}^* & \mathbf{b}^*\end{array}\right]$ (necessary condition for a local minimum [7]). Here, the subdifferential set $\partial J(\mathbf{G}^*,\mathbf{b}^*)$ is given by

$$\partial J(\mathbf{G}^*,\mathbf{b}^*)=\sum_{m=1}^{p}\partial J_m(\mathbf{G}^*,\mathbf{b}^*), \tag{19}$$

where

$$\begin{aligned}\partial J_m(\mathbf{G},\mathbf{b})=\mathbf{Co}\{sign(o_m(l))\mathbf{e}_m\left[\begin{array}{cc}\mathbf{s}_l^T & 1\end{array}\right] \mid \\ l\in\mathscr{I}_m(\mathbf{G},\mathbf{b})\},\end{aligned}$$

$\mathbf{Co}$ is the convex hull operation, $\mathbf{e}_m$ is the m^{th} standard basis vector, $o_m(l)$ is the value of the m^{th} output component at time index l, and

$$\mathscr{I}_m(\mathbf{G},\mathbf{b})=\{l \mid |o_m(l)|=J_m(\mathbf{G},\mathbf{b})\}, \tag{20}$$

i.e., the set of index values for which the maximum magnitude is achieved at the m^{th} component of $\mathbf{o}$. Therefore, a subgradient of $J(\mathbf{G},\mathbf{b})$ can be written as

$$\begin{aligned}&\mathbf{g}(\mathbf{G},\mathbf{b})\\ &=\sum_{m=1}^{p}\sum_{l\in\mathscr{I}_m(\mathbf{G},\mathbf{b})}\lambda_l^{(m)} sign(o_m(l))\mathbf{e}_m\left[\begin{array}{cc}\mathbf{s}_l^T & 1\end{array}\right]\end{aligned} \tag{21}$$

where

$$\sum_{l\in\mathscr{I}_m(\mathbf{G},\mathbf{b})}\lambda_l^{(m)}=1, \text{ and }, \lambda_l^{(m)}\geq 0, \text{ for } l\in\mathscr{I}_m, \quad m=1,\ldots,p. \tag{22}$$

Since there are no constraints on $\mathbf{b}$ parameter, the necessary condition for (local) minima we stated before implies that the last column of the subgradient expression in (21) to be equal to zero, i.e.,

$$\sum_{l\in\mathscr{I}_m(\mathbf{G}^*,\mathbf{b}^*)}\lambda_l^{(m)} sign(o_m(l))=0, \quad m=1,\ldots,p. \tag{23}$$

This equation, together with the fact that $\lambda_l^{(m)}$'s are nonnegative, implies that the magnitudes of the negative and positive peaks of the output should be equal for a minimum point, i.e.,

$$\begin{aligned}&U\|\mathbf{G}^{*(p)}_{m,:}\|_1+L\|\mathbf{G}^{*(n)}_{m,:}\|_1+b_m^*\\ &=L\|\mathbf{G}^{*(p)}_{m,:}\|_1+U\|\mathbf{G}^{*(n)}_{m,:}\|_1-b_m^*,\end{aligned} \tag{24}$$

which further implies

$$b_m^* = \frac{L-U}{2}(|\mathbf{G}^{*(p)}_{m,:}\|_1 - \|\mathbf{G}^{*(n)}_{m,:}\|_1), \quad m = 1,\ldots,p. \tag{25}$$

We now show that among all local minima, perfect separators are the only global minimizers. We first note that for a minimum point, the maximum magnitude of the m^{th} component of $\mathbf{o}$ is given by

$$\begin{aligned} |o_m(l)| &= U\|\mathbf{G}^{*(p)}_{m,:}\|_1 + L\|\mathbf{G}^{*(n)}_{m,:}\|_1 + b_m^*, \quad l \in \mathscr{I}_m(\mathbf{G}^*, \mathbf{b}^*) \\ &= U\|\mathbf{G}^{*(p)}_{m,:}\|_1 + L\|\mathbf{G}^{*(n)}_{m,:}\|_1 \\ &+ \frac{L-U}{2}(|\mathbf{G}^{*(p)}_{m,:}\|_1 - \|\mathbf{G}^{*(n)}_{m,:}\|_1) \\ &= \frac{U+L}{2}(|\mathbf{G}^{*(p)}_{m,:}\|_1 + \|\mathbf{G}^{*(n)}_{m,:}\|_1) \\ &= \frac{U+L}{2}\|\mathbf{G}^*{}_{m,:}\|_1, \end{aligned} \tag{26}$$

where the second equality above follows from (25). We note that

$$\|\mathbf{G}^*{}_{m,:}\|_1 \geq \|\mathbf{G}^*{}_{m,:}\|_2 = 1 \tag{27}$$

with equality if and only if $\mathbf{G}^*{}_{m,:}$ is a Kronecker delta function with single nonzero value. Therefore the global minimum magnitude value $\frac{U+L}{2}$ is achieved only for $\mathbf{G}$ matrices that can be written as,

$$\mathbf{G} = \mathbf{D}\mathbf{P}, \tag{28}$$

where $\mathbf{D}$ is a diagonal matrix with $\{+1,-1\}$ diagonal entries and $\mathbf{P}$ is a permutation matrix. As a result, we conclude that the perfect separators are the only global minima of the proposed optimization problem.

4. ADAPTIVE ALGORITHM

The adaptive algorithm corresponding to the proposed cost function for the bounded domain signals can be developed based on subgradient optimization techniques, following similar steps as in [5]. For the adaptive implementation we assume that M whitened mixture vectors are available and the affine mapping is applied to these mixtures to produce

$$\mathbf{o}^{(i)} = \mathbf{\Theta}^{(i)}\mathbf{x}(k) \qquad k = 1,\ldots,M, \tag{29}$$

at each iteration. We can write the subgradient search based algorithm steps as

$$\mathbf{b}^{(i+1)} = \mathbf{b}^{(i)} - \mu_{\mathbf{b}}^{(i)} \sum_{m=1}^{p} \sum_{l \in I_m^{(i)}} \lambda_l^{(m,i)} sign(o_m^{(i)}(l))\mathbf{e}_m$$

$$\bar{\mathbf{\Theta}}^{(i+1)} = \mathbf{\Theta}^{(i)} - \mu_{\mathbf{\Theta}}^{(i)} \sum_{m=1}^{p} \sum_{l \in I_m^{(i)}} \lambda_l^{(m,i)} sign(o_m^{(i)}(l))\mathbf{e}_m \mathbf{x}_l{}^T$$

$$\mathbf{\Theta}^{(i+1)} = \mathscr{P}\{\bar{\mathbf{\Theta}}^{(i+1)}\},$$

where

- $\mu_b^{(i)}$ and $\mu_{\Theta}^{(i)}$ are step size values used for the subgradient updates,
- $\mathscr{I}_m^{(i)}$ is the set of indices for which maximum magnitude output is achieved at the m^{th} separator output for the i^{th} iteration,
- $\lambda_l^{(m,i)}$ are the convex combination weights that are used to form the subgradient at the i^{th} iteration,
- $\mathscr{P}$ is the operator that projects its input to the set of orthogonal matrices. If $\bar{\mathbf{\Theta}}^{(i+1)}$ has a (full) singular value decomposition given by

$$\bar{\mathbf{\Theta}}^{(i+1)} = \mathbf{U}^{(i+1)}\mathbf{\Sigma}^{(i+1)}\mathbf{V}^{(i+1)^T} \tag{30}$$

then we can define

$$\mathscr{P}\{\bar{\mathbf{\Theta}}^{(i+1)}\} = \mathbf{U}^{(i+1)}\mathbf{V}^{(i+1)^T}. \tag{31}$$

Another option for mapping to the set of orthogonal matrices is to use Gram-Schmidt orthogonalization procedure [2].

The algorithm expressions can be simplified if we choose only one nonzero $\lambda_l^{(m,i)}$ value for each output component and for each iteration. This corresponds to choosing a single time instant from the set of time indexes where the maximum magnitude output is achieved for each output component. In this case, the algorithm can be written as

$$\mathbf{b}^{(i+1)} = \mathbf{b}^{(i)} - \mu_{\mathbf{b}}^{(i)} \sum_{m=1}^{p} sign(o_m^{(i)}(l^{(m,i)}))\mathbf{e}_m, \tag{32}$$

$$\bar{\mathbf{\Theta}}^{(i+1)} = \mathbf{\Theta}^{(i)} - \mu_{\mathbf{\Theta}}^{(i)} \sum_{m=1}^{p} sign(o_m^{(i)}(l^{(m,i)}))\mathbf{e}_m\mathbf{x}^T_{l(m,i)}, \tag{33}$$

$$\mathbf{\Theta}^{(i+1)} = \mathscr{P}\{\bar{\mathbf{\Theta}}^{(i+1)}\}, \tag{34}$$

where $l^{(m,i)}$ is the chosen time index for the i^{th} iteration and the m^{th} output component.

4.1 On the Stationary Points of the Algorithm

It is interesting to analyze the potential convergence points for the proposed algorithm. Based on the algorithm specified in (32)-(33), $\mathbf{G}^*$ is a stationary point of the algorithm if

$$\begin{aligned} \mathbf{G}^* &= \mathscr{P}\{\mathbf{G}^* - \mu' sign(\mathbf{G}^*)\} & (35) \\ &= \mathbf{G}^*\mathscr{P}\{\mathbf{I} - \mu'\mathbf{G}^{*T} sign(\mathbf{G}^*)\}. & (36) \end{aligned}$$

The equivalent condition can be written as

$$\mathscr{P}\{\mathbf{I} - \mu'\mathbf{G}^{*T} sign(\mathbf{G}^*)\} = \mathbf{I}, \tag{37}$$

which further implies that

$$\mathbf{I} - \mu'\mathbf{G}^{*T} sign(\mathbf{G}^*), \tag{38}$$

is a positive definite (for some sufficiently small μ') and $\mathbf{G}^{*T} sign(\mathbf{G}^*)$ is a symmetric matrix.

It is easy to show that $\mathbf{G}$ matrices in (28) corresponding to the perfect separation satisfy this condition:

$$\mathbf{G} = \mathbf{D}\mathbf{P} \Rightarrow sign(\mathbf{G}) = \mathbf{G} \Rightarrow \mathbf{G}^T sign(\mathbf{G}) = \mathbf{I} \tag{39}$$

In addition to the perfect separation points, $\mathbf{G}$ matrices for which the non-zero elements in a given column has constant magnitude, are the stationary points. If β_i represents the magnitude of the non-zero elements in column-i, then

$$sign(\mathbf{G}) = \mathbf{G}\mathbf{B}^{-1} \tag{40}$$

with $\mathbf{B} = diag(\beta_1, \beta_2, \ldots, \beta_p)$. Therefore,

$$\mathbf{G}^T sign(\mathbf{G}) = \mathbf{B}^{-1}, \tag{41}$$

which is a symmetric matrix.

Another interesting group of stationary points is given by

$$\mathbf{G} = \mathbf{I} - 2\frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}}, \tag{42}$$

where $\mathbf{v}$ is a column vector whose nonzero entries have the same magnitude. It can be shown that this special subgroup of Householder reflectors satisfy conditions for the stationary points. If $\mathscr{L}$ represent the number of non-zero entries in $\mathbf{v}$, when

- $\mathscr{L} = 1$: $\mathbf{G}$ is a diagonal matrix and therefore the stationary point condition is clearly satisfied.
- $\mathscr{L} = 2$: $\mathbf{G}$ is a perfect separation matrix and therefore satisfies the stationary point condition.
- $\mathscr{L} > 2$: Without loss of generality, if we assume that the first L of the entries of $\mathbf{v}$ are non-zero and the other elements are zero, we can write

$$\mathbf{v} = \begin{bmatrix} \frac{\rho}{\sqrt{\mathscr{L}}} \\ \mathbf{0} \end{bmatrix} \tag{43}$$

where $\rho \in \Re^{\mathscr{L}\times 1}$ is a vector with elements in $\{1, -1\}$. Based on this definition, $\mathbf{G}$ would be equivalent to

$$\begin{aligned} \mathbf{G} &= \mathbf{I} - 2\mathbf{v}\mathbf{v}^T & (44)\\ &= \mathbf{I} - \frac{2}{\mathscr{L}}\begin{bmatrix} \rho\rho^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & (45)\\ &= \begin{bmatrix} \frac{\mathscr{L}-2}{\mathscr{L}}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \frac{2}{\mathscr{L}}\begin{bmatrix} (\rho\rho^T - \mathbf{I}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & (46) \end{aligned}$$

Therefore, $sign(\mathbf{G})$ can be written as

$$sign(\mathbf{G}) = \mathbf{I} - \begin{bmatrix} (\rho\rho^T - \mathbf{I}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{47}$$

As a result, we can write

$$\mathbf{G}^T sign(\mathbf{G}) = \begin{bmatrix} 2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \frac{\mathscr{L}-4}{\mathscr{L}}\mathbf{v}\mathbf{v}^T, \tag{48}$$

which is a symmetric matrix, and consequently we deduce that the corresponding $\mathbf{G}$ is a stationary point.

One important observation is that the stationary points outlined above are also the stationary points of the Kurtosis-cost based algorithm using symmetric orthogonalization [8]. This observation also contributes to the view that infinity norm minimization based finite support approach is a complementary approach to the higher-norm maximization approaches tied to Donoho's objective functions [9].

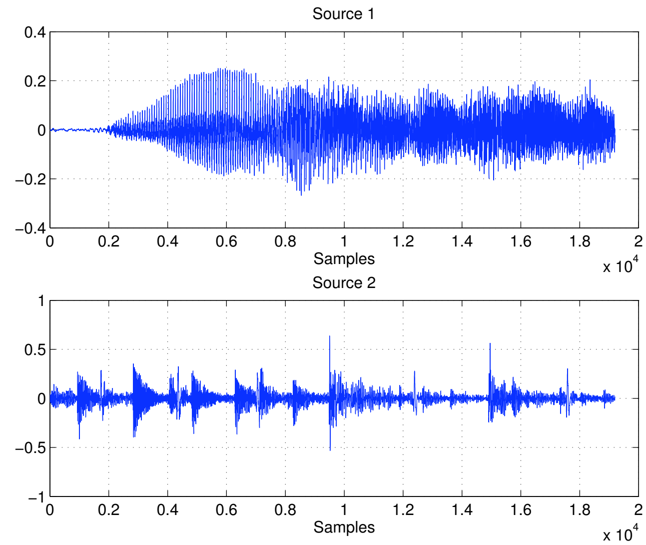

Figure 1: Source signals for the Example.

5. EXAMPLE

In this section we illustrate the performance of the proposed algorithm through a numerical simulation example. We consider the 2×2 instantaneous audio separation example in [10, 11]. For this example the sources are as shown in Figure 1.

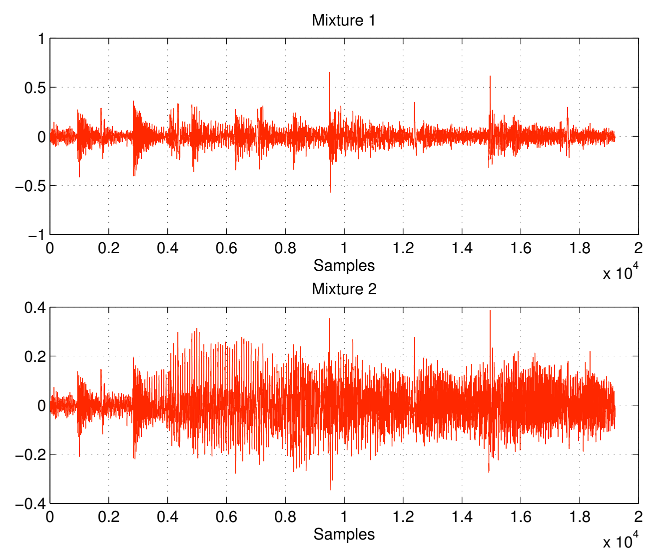

Figure 2: Mixture signals for the Example.

The instantaneous mixtures corresponding to these sources are shown in Figure 2.

The proposed algorithm is applied to a short data window (50 samples), where the data window is changed to the next nonoverlapping window at each iteration, i.e., a moving window approach is used. The step sizes used at the k^{th} iteration is

$$\mu_{m,\Theta}^{(k)} = \frac{5}{k\|x_{l_{m,i}}\|} \qquad \mu_{m,\mathbf{b}}^{(k)} = \frac{20}{k} \tag{49}$$

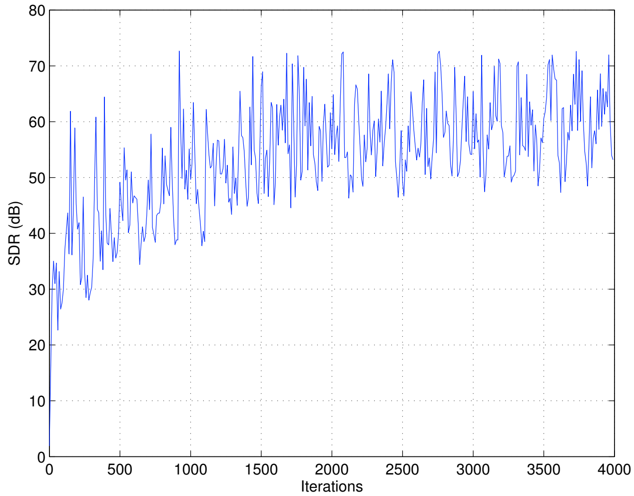

Figure 3: SDR Performance of the Proposed Algorithm.

The SDR performance of the algorithm which is evaluated using the "BSS Evaluation Toolbox" [10, 11], as a function of iterations is shown in Figure 3. As can be seen from this figure, the algorithm achieves high separation performance in small number of iterations.

6. CONCLUSION

We introduced a simple BSS approach that exploits two simple assumptions about the sources, namely, independence and domain boundedness, which are fairly reasonable assumptions in real applications. The proposed cost function converts the support length minimization problem into the magnitude peak minimization problem. The corresponding adaptive algorithm has a relatively simple update rule. We also provided an analysis for the stationary points of the proposed algorithm where it is shown that the Kurtosis-based (symmetrical) BSS approach and the proposed approach share the same set of stationary points.

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