Let γ^μ, β, $u_s(\mathbf{p})$ and $v_s(\mathbf{p})$ be as defined in Sections 36-38 of Srednicki, and

$$\psi_{1,s}(\mathbf{p}) := u_s(\mathbf{p}), \qquad \psi_{2,s}(\mathbf{p}) := v_s(\mathbf{p}),$$

$$\phi_{1,s}(\mathbf{p}) := \frac{1}{2m}\,\bar{u}_s(\mathbf{p})^\dagger, \qquad \phi_{2,s}(\mathbf{p}) := -\frac{1}{2m}\,\bar{v}_s(\mathbf{p})^\dagger,$$

$$\beta_+ := 2m\sum_{s=\pm}\sum_{j=1}^{2}\phi_{j,s}(\mathbf{p})\phi_{j,s}(\mathbf{p})^\dagger, \qquad \mathbf{A} := -\gamma^\mu p_\mu.$$

Solve the following problems (In your solution you may use any mathematical identity that is derived in class. If you need any other result, you must give its derivation before using it.)

1 (10 points) Show that $\beta = (2m)^{-1}\sum_{s=\pm}\left[u_s(\mathbf{p})u_s(\mathbf{p})^\dagger - v_s(\mathbf{p})v_s(\mathbf{p})^\dagger\right]$.

2 (10 points) Obtain an explicit expression for β_+ in terms of the boost generators K^i. Hint: Use Eqs. 38.12 of Srednicki.

3 (10 points) Show that $\mathbf{A}^\dagger = \beta_+\mathbf{A}\beta_+^{-1}$.

4 (5 points) Show that there is a positive matrix ρ such that $\rho^2 = \beta_+$.

5 (10 points) Let for any 4×4 matrix $\mathbf{M}$, $\widehat{\mathbf{M}} := \rho\,\mathbf{M}\,\rho^{-1}$. Show that $\widehat{\mathbf{A}}$ is Hermitian.

6 (5 points) Show that $\widehat{\gamma}^0 = e^{-2i\eta\hat{\mathbf{p}}\cdot\mathbf{K}}\gamma^0$, where η and $\hat{\mathbf{p}}$ are the ones appearing in Eqs. 38.12 of Srednicki.

7 (20 points) Show that $\dfrac{\partial}{\partial\eta}\widehat{\gamma}^j = \hat{p}^j\widehat{\gamma}^0$.

8 (20 points) Use the identity given in Problem 6 to solve the equation given in Problem 7 and find an explicit expression for $\widehat{\gamma}^j$.

9 (10 points) Use your response to Problem 8 to find an explicit expression for $\widehat{A}$. Simplify this expression as much as possible.