Phys 401: Midterm Exam 1

October 13, 2018

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 2.5 hours.
- You must show the details of all your work. Illegible and ambiguous explanations and calculations will lead to deductions from your grade.
- You may use the option of grading your own work. If your estimated grade differs from your actual grade by less than 10 points, you will be given the higher of the two.

Estimated Grade:	
Actual Grade:	
Adjusted Grade:	

Problem 1 (10 points) Give the definition of the states, phase space, and observables of a classical system in the Lagrangian formulation of classical mechanics.

Problem 2 (10 points) Consider an observable of a classical system in the Hamiltonian formulation of classical mechanics. Let H be its Hamiltonian and O be an observable that does not depend explicitly on time, i.e., $O = O(q,p)$, where $q := (q_1, q_2, \cdots, q_n)$ represents the configuration space coordinates and $p := (p_1, p_2, \cdots, p_n)$ represents the conjugate momenta. Use Hamilton's equations of motion to show that $O(t) := O(q(t), p(t))$ satisfies $\frac{d}{dt}O(t) = \{O(t), H\}_{\rm PB}$.

Problem 3 Consider a particle of mass m and electric charge q that moves in a three dimensional Euclidean space and interacts with a magnetic field $\vec{B}$. The dynamics of this particle is determined by the Lagrangian: $L = \frac{m\vec{v}^2}{2} + \frac{q}{c}\vec{A}\cdot\vec{v}$, where $\vec{v} = \dot{\vec{x}}$ is the velocity of the particle, $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$ is its position, $\hat{i}$, $\hat{j}$, and $\hat{k}$ are respectively unit vectors along the x-, y-, and z-axes, c is the speed of light in vacuum, $\vec{A}$ is the vector potential given by: $\vec{A} := \frac{B_0}{2}(-y\hat{i} + x\hat{j})$, that gives the magnetic field according to $\vec{B} = \vec{\nabla}\times\vec{A}$, and B_0 is a constant parameter.

3.a (15 points) Derive the equations of motion for the particle.

3.b (20 points) Find the explicit solution of the equations of motion that satisfy the initial conditions:

$$x(0) = \ell, \qquad y(0) = 0, \qquad z(0) = \ell$$
$$\dot{x}(0) = 0, \qquad \dot{y}(0) = b, \qquad \dot{z}(0) = a,$$

where ℓ and a are real constants, and $b := -\frac{qB_0\ell}{mc}$. What is the shape of the trajectory of the particle in the configuration space?

Warning: To get proper credit, you must simplify your response as much as possible.

Problem 4 (15 points) Find the Hamiltonian for the system described in Problem 3 and use it to obtain the rate of change of the z-component of the angular momentum of the particle in time.

Problem 5 Consider a system with phase space $\mathbb{R}^2$ and let $\mathscr{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathscr{T}(q,p) = (\tilde{q},\tilde{p})$ where

$$\tilde{q} = \alpha(t)q + \beta(t)p,$$
$$\tilde{p} = \gamma(t)q + \delta(t),$$

and $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\delta(t)$ are smooth real-valued functions.

5.a (5 points) Find the necessary and sufficient condition on $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\delta(t)$ such that the $\mathscr{T}$ is a canonical transformation.

5.b (15 points) Determine how $\mathscr{T}$ transforms the Hamiltonian of a free particle of mass m, i.e., suppose that $H = p^2/2m$ and determine the explicit form of the transformed Hamiltonian $\tilde{H}$, which is a function of $\tilde{q}, \tilde{p}$, and t.

6 (10 points) Consider a particle of mass m that moves in a plane with position $\vec{x} = (x_1, x_2)$ and momentum $\vec{p} = (p_1, p_2)$. Suppose that the Hamiltonian of the system is a function of $x_1 + 3x_2$, and $2p_1 - p_2$, i.e., there is a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $H = f(x_1 + 3x_2, 2p_1 - p_2)$. Show that $(x_1 + 2x_2)(3p_1 - p_2)$ is a conserved quantity.