Math 503: Midterm Exam 1

Fall 2011

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have One hour and forty five minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) := 1 + \sqrt{x - 1}$.

1.a) Determine the domain and range of f (No justification is necessary.) (5 points)

1.b) Show whether f is a one-to-one function. (5 points)

1.c) Find the image of the interval $[-1, 3]$ under f. (5 points)

1.d) Find the inverse image of the interval $[-1, 3]$ under f. (5 points)

Problem 2. Compute the real and imaginary parts of $e^{-2\imath z}$ for $z := \frac{\pi}{4} - 2\imath$.   (10 points)

Problem 3) Show whether the following polynomials form a linearly independent subset of the vector space $\mathcal{P}(\mathbb{R}, \mathbb{R})$ of all polynomials $p : \mathbb{R} \to \mathbb{R}$.   (10 points)

$$p_1(x) := 3x^3 - x^2 - 2x + 1, \quad p_2(x) := -2x^3 - 2x^2 + x - 1, \quad p_3(x) := -8x^2 - x - 1.$$

Problem 4) Let $\mathbf{b}_1 := (\imath, 1)$, $\mathbf{b}_2 := (1, 0)$, and $L : \mathbb{C}^2 \to \mathbb{C}^2$ be the linear operator defined by

$$L(w, z) := \big((-1 + 2\,\imath)w + (2 + \imath)z, (1 + \imath)w + (1 - \imath)z\big).$$

4.a) Show that the null space of L is the span of $\{\mathbf{b}_1\}$. (10 points)

4.b) Show that $\{\mathbf{b}_1, \mathbf{b}_2\}$ is a basis of $\mathbb{C}^2$. (10 points)

4.c) Determine $L\mathbf{b}_2$ and show that range of L is the span of $\{L\mathbf{b}_2\}$. (10 points)

Problem 5) Let $L : \mathbb{C}^2 \to \mathbb{C}^2$ be the linear operator defined by

$$L(w, z) := ((-1 + 2\imath)w + (2 + \imath)z, (1 + \imath)w + (1 - \imath)z),$$

as in Problem 4. Discuss the existence and uniqueness of the solution of following linear equations.

5.a) $L(w, z) = (0, 0)$. (5 points)

5.b) $L(w, z) = (1, -\imath)$. (10 points)

Problem 6) Let $L : \mathbb{C}^2 \to \mathbb{C}^2$ be the linear operator defined by

$$L(w, z) := ((-1 + 2\,\imath)w + (2 + \imath)z, (1 + \imath)w + (1 - \imath)z),$$

as in Problem 4. Give the matrix representation of L in the basis consisting of $\mathbf{b}_1 := (\imath, 1)$ and $\mathbf{b}_2 := (1, 0)$. (15 points)