Math 503: Midterm Exam # 1

Fall 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 80 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let f_1, f_2, f_3 be the functions defined by: $\forall x \in \mathbb{R}$, $f_1(x) := 1$, $f_2(x) := e^{ix}$, and $f_3(x) := xe^{ix}$. Prove that these functions are linearly independent. (15 points)

Problem 2. Let V be the complex inner product space of polynomials $p : [0,1] \to \mathbb{C}$ with the inner product $\langle \cdot, \cdot \rangle : V^2 \to \mathbb{C}$ defined by

$$\forall f, g \in V, \quad \langle f, g \rangle := \int_0^1 x^2 \, \overline{f(x)} \, g(x) \, dx,$$

and $W := \mathrm{Span}(\{p_1, p_2\})$ where $p_1, p_2 \in V$ are defined by: $\forall x \in [0,1]$, $p_1(x) := 1$ and $p_2(x) := x$. Let $L : W \to W$ be defined by

$$\forall f \in W, \forall x \in [0,1], \quad (Lf)(x) := \frac{1}{x^2} \int_0^x t \, f(t) \, dt.$$

2.a) Find the matrix representation of L in the basis $\{p_1, p_2\}$. (15 points)

2.b) Solve the eigenvalue problem for L in W, i.e., find the eigenvalues and eigenvectors of L. (15 points)

2.c) Determine whether L is a self-adjoint operator. You must justify your response. (10 points)
Hint: You may use your response to part 2.b.

Problem 3. Use the fact that $y_1(x) := x^2$ solves $y'' - 2x^{-2}y = 0$ to find the general solution of $y'' - 2x^{-2}y = x$. (25 points)

Problem 4. Find a solution of $x^2y'' + 4xy' - (x^2 - 2)y = 0$. (25 points)