Math 503, Fall 2006
Assignment for Nov. 30 - Dec. 04

- Read pages 782-796 of Riley-Hobson-Bence.
- Solve Problems 22.1, 22.2, 22.20, 22.22, on pages 797-799 of Riley-Hobson-Bence.
- Solve the following problems.

1. Find the stationary points of the following functionals.

$$\begin{aligned} \mathcal{F}[y(x)] &= \int_a^b \sqrt{1 + \frac{y'^2}{y^2}}\, dx, \\ \mathcal{G}[y(x)] &= \int_a^b \frac{\sqrt{1+y'^2}}{1+y}\, dx, \end{aligned}$$

2. Let S be the surface of revolution of the curve $z = x^2$ about z-axis. Find the differential equation determining the geodesics on S and obtain its solution.

3. Let $\mathcal{F}$ and $\mathcal{G}$ be functionals. Prove that

$$\frac{\delta}{\delta y(t)} \left(\mathcal{F}[y(s)]\mathcal{G}[y(s)]\right) = \frac{\delta \mathcal{F}[y(s)]}{\delta y(t)}\, \mathcal{G}[y(s)] + \mathcal{F}[y(s)]\, \frac{\delta \mathcal{G}[y(s)]}{\delta y(t)}.$$

4. Show that if $\mathcal{F}[y(x)] := \int_a^b y'^2(x)\ dx$. The second functional derivative of $\mathcal{F}[y(s)]$ is given by

$$\frac{\delta}{\delta y(u)} \frac{\delta}{\delta y(t)}\, \mathcal{F}[y(s)] = -\frac{\partial^2}{\partial t^2}\, \delta(t-u).$$