Math 450/586: Quiz # 5

Fall 2009

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 60 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask, 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Give the definition of the following objects. (15 points)

1.a) An integral curve of a vector field:

1.b) A geodesic in a C^∞-manifold with an affine connection:

1.c) A Riemannian manifold:

2. Let $X := x^2\frac{\partial}{\partial x^1} + x^1\frac{\partial}{\partial x^2}$ be a vector field on $\mathbb{R}^2$, where (x^1, x^2) are coordinates of points in $\mathbb{R}^2$ in a coordinate system. Find the Lie derivative of the following fields along X.

2.a) The scalar field $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x^1, x^2) = x^1 - x^2$: (10 points)

2.b) The vector field $Y = x^2\frac{\partial}{\partial x^1} - x^1\frac{\partial}{\partial x^2}$: (10 points)

2.c) The one-form $\omega = x^2dx^1 - x^1dx^2$: (15 points)

<u>Hint:</u> Recall that $\mathcal{L}_X dx^i = X^i_{\ ,j}\, dx^j$.

3. Let the Christoffel symbols for an affine connection on $\mathbb{R}^2$ be given by

$$\Gamma^1_{11} = x^1, \quad \Gamma^1_{12} = \Gamma^1_{21} = x^1 + x^2, \quad \Gamma^1_{22} = -2x^2,$$

$$\Gamma^2_{11} = x^2, \quad \Gamma^2_{12} = \Gamma^2_{21} = x^1 - x^2, \quad \Gamma^2_{22} = 0,$$

$\gamma : [0,1] \to \mathbb{R}^2$ be the curve defined by $\gamma(t) = (t,t)$, and $V_0 = \frac{\partial}{\partial x^1}$ belong to the tangent space of $\mathbb{R}^2$ at $\gamma(0)$. Find the components V^i of the tangent vector $V^i \frac{\partial}{\partial x^i}$ obtained by parallel transportation of the vector V_0 along γ to the point $\gamma(1)$. (50 points)

<u>Hint:</u> The equation for parallel transportation is $\dfrac{dV^i}{dt} + \Gamma^i_{jk} \dfrac{dx^j}{dt} V^k = 0.$