Math 303: Quiz # 1

Fall 2013

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 35 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time no question will be answered.)

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1. Find the real and imaginary parts of all possible values of $\sin(\frac{\pi}{2} + i\ln 2)$. (8 points)

$$\sin\left(\tfrac{\pi}{2} + i\ln 2\right) = \frac{1}{2i}\left[e^{i(\frac{\pi}{2} + i\ln 2)} - e^{-i(\frac{\pi}{2} + i\ln 2)}\right]$$

$$= \frac{1}{2i}\left[\underbrace{e^{i\frac{\pi}{2}}}_{=\,i}\ \underbrace{e^{-\ln 2}}_{e^{\ln 2^{-1}} \,=\, \frac{1}{2}} - \underbrace{e^{-i\frac{\pi}{2}}}_{-i}\ \underbrace{e^{\ln 2}}_{2}\right]$$

$$= \frac{1}{2i}\left[\frac{i}{2} + 2i\right] = \frac{1}{2}\left(2 + \frac{1}{2}\right) = 1 + \frac{1}{4} = \frac{5}{4}$$

So $\sin(\frac{\pi}{2} + i\ln 2)$ has a single real value which is $\frac{5}{4}$. Its imaginary part is zero.

$$\begin{cases} \mathrm{Re}\left[\sin\left(\frac{\pi}{2} + i\ln 2\right)\right] = \frac{5}{4} \\ \mathrm{Im}\left[\sin\left(\frac{\pi}{2} + i\ln 2\right)\right] = 0 \end{cases}$$

2. Calculate the first two nonzero terms in the Taylor series expansion of $\ln x$ about $x = 1$ and use it to find an approximate value for $\ln(1.1)$. (8 points)

$$(\ln x)' = \frac{1}{x} \qquad x > 0$$

$$(\ln x)'' = -\frac{1}{x^2} \qquad x > 0$$

$$\ln x = \ln(1) + \ln'(1)(x-1) + \frac{\ln''(1)}{2}(x-1)^2 + \cdots$$

$$= 0 + (x-1) - \frac{1}{2}(x-1)^2 + \cdots$$

$$\ln(1.1) = (1.1-1) - \frac{1}{2}(1.1-1)^2 + \cdots$$

$$\approx 0.1 - \frac{1}{2}(0.1)^2 = 0.1 - \frac{1}{200}$$

$$\text{or } \frac{5}{1000}$$

$$\approx 0.095$$

3. Find the stationary points of the function $f(x, y) = x^3y + xy^3 + y^5 - 8y + 5$ and determine if they are local minimum, local maximum, or saddle points. Justify your response. (14 points)

$$f_x = 3x^2y + y^3 = 0 \Rightarrow y(3x^2 + y^2) = 0$$

$$\Downarrow$$

$$\boxed{y = 0}$$

$$f_y = x^3 + 3xy^2 + 5y^4 - 8 = 0 \quad \hookrightarrow \quad x^3 = 8$$

$$\Downarrow$$

$$\boxed{x = 2}$$

$\boxed{\text{(2,0) is the only stationary point of } f(x,y)}$

$$f_{xx} = 6xy, \quad f_{xy} = 3x^2, \quad f_{yy} = 6xy + 20y$$

$$H(2,0) = \begin{bmatrix} f_{xx}(2,0) & f_{xy}(2,0) \\ f_{xy}(2,0) & f_{yy}(2,0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 12 \\ 12 & 0 \end{bmatrix}$$

$$\det H(2,0) = -144 < 0 \Rightarrow \boxed{\text{(2,0) is a saddle point.}}$$