Math 303: Final Exam

Fall 2008

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have two hours (120 minutes).
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

To be filled by the grader:

Problem 1:	
Problem 2:	
Problem 3:	
Problem 4:	
Problem 5:	
Problem 6:	
Total Grade:	

Problem 1. Let a, b, and c be real numbers satisfying $a^2 + 2b^2 + 3c^2 = 1$.

1.a) Find the values of a, b, and c for which $a+b+c$ has the largest possible value. (12 points)

1.b) Find the values of a, b, and c for which $a+b+c$ has the smallest possible value. (3 points)

Problem 2. Let $B := \{(x, y, z) \in \mathbb{R}^3 \mid 0 \leq z \leq \sqrt{1 - x^2 - y^2}\ \}$, S be the boundary of B, $\mathbf{n}$ be the unit normal outward vector to S, and $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field defined by $\mathbf{F}(x, y, z) := (xz + y^3)\mathbf{i} + x^5\mathbf{j} + (z - y^3)\mathbf{k}$, where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along the x-, y-, z-axes, respectively. Evaluate the following surface integral. (15 points)

$$\iint_S \mathbf{F} \cdot \mathbf{n}\ d\sigma$$

Problem 3. Let $\delta(t)$ denote the one-dimensional Dirac delta-function, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ be the unit vectors along the x-, y-, z-axes in a Cartesian coordinate system, $\mathbf{r} := x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and $r := |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$. Compute the integral of $\dfrac{e^{-r}\mathbf{r} \cdot \nabla\delta(r-1)}{r}$ over all the space $\mathbb{R}^3$, i.e., evaluate the volume integral $\displaystyle\iiint_{\mathbb{R}^3} \frac{e^{-r}\mathbf{r} \cdot \nabla\delta(r-1)}{r}\, dV.$ (15 points)

Hint: First calculate $\nabla\delta(r-1)$ and use the properties of the delta function to simplify the integrand. Then perform the volume integral in the spherical coordinates.

Problem 4. Let (r, θ, z) be the cylindrical coordinates in $\mathbb{R}^3$ such that the cartesian coordinates x and y are related to r and θ according to $x = r\cos\theta, y = r\sin\theta$. Then the equation $r = 1 + \cos\theta$ defines a cylinder S. Find the equation giving the geodesics (curves of minimum length) on this cylinder. (15 points)

Hint: Parameterize the curves on S by θ and express the length of such a curve as an integral over θ. Note that on such a curve x and y are already functions of θ. You just need to assume that z is also a function of θ, and find the form of this function that makes the length of the curve minimum.

Problem 5. Calculate the Fourier transform $\tilde{f}(k)$ of $f(x) := \frac{1}{(x-1)(x-2)}$ for $k \geq 0$. (25 points)
Hint: Use residue theorem to evaluate the integral giving $\tilde{f}(k)$.

Problem 6. Evaluate $\displaystyle\int_0^\infty \frac{t\sin(\pi t)}{1+t^2}\,dt$ using the residue theorem. (15 points)