Math 303: Midterm Exam 2

Fall 2008

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have One hour and 45 minutes (105 minutes).
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. State and prove Green's theorem in plane for a vector field $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ defined on the region: $D := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1.\}$. (15 points)

Problem 2. Find $g(x)$ such that $e^{-|x|}$ is a solution of $y'' + y = g(x)$. (10 points)

Problem 3. Find the real Fourier series for the following function. (15 points)

$$f(x) = \begin{cases} \sin x & \text{for} \quad -\pi \leq x < 0, \\ \cos x & \text{for} \quad 0 \leq x < \pi, \end{cases} \qquad f(x + 2\pi) = f(x).$$

Hint: $2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$, $2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$, $2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$.

Problem 4. Let $f(x)$ denote the inverse Fourier transform of $e^{-|k|}$. Find the Fourier transform of $(x^2+1)e^{-|x|}f(x)$.   (15 points)

Problem 5. A particle moves in the x-y plane in such a way that its speed is given by its distance from the origin, i.e., $r := \sqrt{x^2 + y^2}$. Determine the path the particle should take to go from the $(1, 0)$ to $(0, 1)$ such that the travel time is minimized. (25 points)

Hint: Use polar coordinates (r, ϕ) where the line element $d\ell$ satisfies $d\ell^2 = dr^2 + r^2 d\phi^2$. Note that the speed of the particle is defined to be $d\ell/dt$.

Problem 6. Let $v(x, y) := y^3 - 3x^2y + y$.

a) Show that v is a solution of the Laplace equation. (5 points)

b) Find an analytic function $f : \mathbb{C} \to \mathbb{C}$ such that $v(x, y)$ is the imaginary part of $f(x + iy)$ and $f(0) = 1$. Give an explicit formula for $f(z)$. (15 points)