Solutions

Name:

Math 303: Quiz # 1

Fall 2004

- You have 35 minutes.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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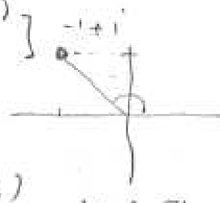
1. Find the polar representation of all values of $\sqrt[i]{-1+i}$. (5 points)

let $z = \sqrt[i]{-1+i}$

$z = (-1+i)^{\frac{1}{i}}$ $\quad |-1+i| = \sqrt{2}$ $\quad \arg(-1+i) = \tan^{-1}(-1) = \frac{3\pi}{4}$

$\Rightarrow z = e^{\frac{1}{i}\ln(-1+i)}$ $\quad \ln(-1+i) = \ln\left[\sqrt{2}\, e^{i(\frac{3\pi}{4}+2k\pi)}\right]$



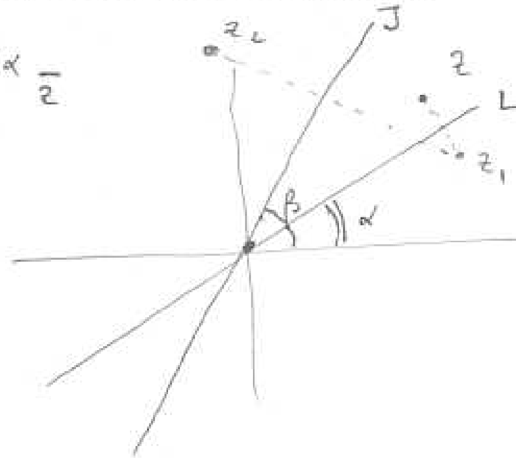
$= \ln\sqrt{2} + i(\frac{3\pi}{4}+2k\pi)$

$= \frac{1}{2}\ln 2 + i(\frac{3\pi}{4}+2k\pi)$ $\quad k \in \mathbb{Z}$

$\Rightarrow z = e^{\frac{\ln 2}{2i}}\, e^{\frac{3\pi}{4}+2k\pi}$

$= e^{(\frac{3}{4}+2k)\pi}\, e^{-\frac{i\ln 2}{2}}, \quad k \in \mathbb{Z}$

2. A point z in complex plane is reflected first about the line L that passes through 0 and makes an angle α with positive real-axis (x-axis) and then reflected about another line J that also passes through 0 and makes an angle β with positive real-axis. Show that the second reflection gives a point that can be obtained from z by a rotation about 0. What is the angle of rotation? (7 points)



$z_1 = R(\alpha)\overline{R(-\alpha)z} = e^{i\alpha}\overline{(e^{-i\alpha}z)} = e^{2i\alpha}\bar{z}$

rotation by α

$z_2 = R(\beta)\overline{R(-\beta)z_1} = e^{2i\beta}\bar{z}_1$

$= e^{2i\beta}\overline{(e^{2i\alpha}\bar{z})}$

$= e^{2i\beta}e^{-2i\alpha}z$

$= e^{i[2(\beta-\alpha)]}z$ $\quad$ clearly $R[2(\beta-\alpha)] = e^{i[2(\beta-\alpha)]}$

The angle of rotation $= 2(\beta-\alpha)$.

3. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^4 + xy + y^4$. Determine whether $(0,0)$ is a local extremum point. If it is, specify whether it is a local maximum or minimum point. If it is not, explain why. (8 points)

3)

$f(x) = x^4 + xy + y^4$

$f_x = 4x^3 + y$, $f_y = x + 4y^3$

$f_{xx} = 12x^2$, $f_{xy} = 1$, $f_{yy} = 12y^2$

$f_x(0,0) = f_y(0,0) = 0$

$f_{xx}(0,0) = f_{yy}(0,0) = 0$, $f_{xy}(0,0) = f_y(0,0) = 1$

So Hessian at (0,0) $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$0 = \det \begin{pmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{pmatrix} \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\Rightarrow$ eignvalues have opposite sign so, (0,0) is <u>not</u> a local extremum point.