Math 208: Midterm Exam 1

Spring 2010

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 80 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. State Completeness Axiom for $\mathbb{R}$ and use it to prove: $\forall r \in \mathbb{R}^+, \exists n \in \mathbb{N}, r < n$. (20 points)

Problem 2. Give the definition of a sequentially compact subset of $\mathbb{R}$ and state The Sequential Compactness (Bolzano-Weierstrass) theorem. (10 points)

Problem 3. Prove that every monotone increasing sequence in $\mathbb{R}$ that has a convergent subsequence converges. (20 points)

Problem 4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, $a, b \in \mathbb{R}$ and $a < b$ Prove that the image of $[a, b]$ under f is bounded above. (20 points)

Problem 5. Let I be an open interval and $f : I \to \mathbb{R}$ be a differentiable function with domain I. Suppose that $\forall x \in I,\ f'(x) > 0$. Prove that f is strictly increasing. (15 points)

Problem 6. Let $\forall n \in \mathbb{N}$, $f_n : [0,1] \to \mathbb{R}$ be the function defined by $\forall x \in [0,1]$, $f_n(x) := \dfrac{x}{nx+1}$. Prove that $\{f_n\}$ converges to the (constant function) zero uniformly. (15 points)