Solutions:

Math 207: Quiz # 1B

Fall 2004

- You have 35 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Determine which of the following series converges. Explain your reasoning.

1.a)

$$\sum_{n=0}^{\infty} \frac{\ln(n+1)}{(n+1)}. \qquad \text{(3 points)}$$

Hint: Use the integral test.

let $f(x) = \frac{\ln(x+1)}{x+1}$, $I = \int_0^\infty f(x)\,dx = \int_0^\infty \frac{\ln(x+1)\,dx}{x+1}$

let $u = \ln(x+1) \Rightarrow du = \frac{dx}{x+1}$

$\Rightarrow \int f(x)\,dx = \int u\,du = \frac{u^2}{2} + c$ So $I = \frac{[\ln(x+1)]^2}{2}\Big|_0^\infty = \infty$

So the series diverges.

1.b)

$$\sum_{n=0}^{\infty} (-1)^n \ln(n+1) \qquad \text{(2 points)}$$

This series diverges because $(-1)^n \ln(n+1) \not\to 0$.

2. Use the power series expansion of e^{t^2} about $t=0$ to estimate the value of

$$f(x) = \int_0^x e^{t^2}\,dt$$

at $x = 0.1$. (7 points)

$$e^{t^2} = 1 + t^2 + \frac{(t^2)^2}{2!} + \frac{(t^2)^3}{3!} + \cdots$$

$$f(x) = \int_0^x e^{t^2}\,dt = t + \frac{t^3}{3} + \frac{t^5}{5\times 2} + \frac{t^7}{6\times 7} + \cdots \Big|_0^x$$

$$= x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \cdots$$

$$f(0.1) \sim 0.1 + \frac{(0.1)^3}{3} + \frac{(0.1)^5}{10}$$

$$\sim 0.1 + \frac{1}{3000} + \frac{1}{10^6}.$$

$$\sim 0.100000 + 0.000333 + 0.000001$$

$$\sim 0.1000334$$

$$\frac{1}{3000} = 0.000333\ldots$$

3. Let $u = 1 - \sqrt{3}i$ and $w = \sqrt{3} - i$. Find the modulus $|z|$, the principal argument $\arg_P(z)$, the real part $\mathrm{Re}(z)$, and the imaginary part $\mathrm{Im}(z)$ of

$$z = \frac{u^2}{\bar{w}^3}. \qquad \text{(8 points)}$$

Hint: First determine the polar form of u and w.

$|u| = \sqrt{1+3} = 2, \qquad |w| = \sqrt{3+1} = 2$

$\arg_p(u) = \tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3}) = 2\pi - \frac{\pi}{3}$

$= \frac{5\pi}{3}$

$\arg(w) = \tan^{-1}(-\frac{1}{\sqrt{3}}) = -\tan^{-1}(\frac{1}{\sqrt{3}}) = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\Rightarrow \quad u = 2e^{i\frac{5\pi}{3}}, \qquad w = 2e^{i\frac{11\pi}{6}}$

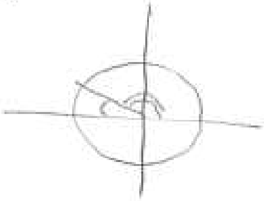
$\Rightarrow \quad \bar{w} = 2e^{-\frac{11\pi}{6}}$

$\Rightarrow \quad z = \dfrac{4 \cdot e^{i\frac{10\pi}{3}}}{8\, e^{-i\frac{11\pi}{2}}} = \frac{1}{2} e^{i(\frac{10}{3} + \frac{11}{2})\pi} = \frac{1}{2} e^{i(\frac{20+33}{6})\pi}$

$\left(\frac{20+33}{6} = \frac{53}{6} = \frac{48+5}{6} = 8 + \frac{5}{6} \right) \qquad = \frac{1}{2} e^{i\frac{5\pi}{6}}$

$\Rightarrow \quad z = \frac{1}{2} e^{\frac{5\pi i}{6}} = \frac{1}{2}\left[\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6})\right]$

so $|z| = \frac{1}{2}, \qquad \arg_p(z) = \frac{5\pi}{6}$



$\mathrm{Re}(z) = \frac{1}{2}\cos(\frac{5\pi}{6}) = -\frac{1}{2}\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{4}$

$\mathrm{Im}(z) = \frac{1}{2}\sin(\frac{5\pi}{6}) = +\frac{1}{2}\sin(\frac{\pi}{6}) = \frac{1}{4}$