Name, Last Name: **Grade:**

Student No:

Math 103: Quiz # 5

Spring 2007

You have 45 minutes.

1. Let A be a nonempty set, $a \in A$, $B \subseteq A$, $\sim \subseteq A^2$ be an equivalence relation, and $\preceq \subseteq A^2$ be a partial ordering relation.

1.a) Give the definition of the equivalence class of a; (5 points)

1.b) Give condition(s) under which $\preceq$ is a total ordering; (5 points)

1.c) Give the definition of a minimal element of A; (5 points)

1.d) Give the definition of the supremum of B in A; (5 points)

1.e) Explain what it means for A to have the supremum property. (5 points)

2. Let A be a nonempty set and $\sim \subseteq A^2$ be an equivalence relation. Prove that $\forall a, b \in A$, $(a \sim b) \Rightarrow ([a] = [b])$. (20 points)

3. Let A be a nonempty set and $\{A_\alpha\}_{\alpha \in \mathfrak{A}}$ be a partition of A. Show that

$$\sim := \{(a, b) \in A^2 \mid \exists \alpha \in \mathfrak{A},\ (a \in A_\alpha) \wedge (b \in A_\alpha)\ \}$$

is an equivalence relation. (25 points)

4. Let $(A, \preccurlyeq)$ be a poset, $B \subseteq A$, and u be the infimum of B in A. Show that B has no other infimums in A, i.e., u is unique. (15 points)

5. Let $A := \{1, 2, 3\}$.

(a) Find a partial ordering relation $\preccurlyeq$ on A such that the poset $(A, \preccurlyeq)$ has no least or greatest elements. (5 points)

(b) Find the maximal and minimal elements of the poset you constructed in part (a). (5 points)

(c) List all chains of the poset you constructed in part (a). (5 points)