

Name, Last Name:

Grade:

Student No:

Math 103: Quiz # 2

Spring 2007

You have 45 minutes.

1.a) Give the statement of Division Algorithm. (5 points)

1.b) Give the statement of Induction Axiom. (5 points)

1.c) Explain the difference between ordinary and complete induction. (5 points)

2. Use the definition of the “absolute value of a real number” and the method of proof by cases to prove “ $\forall x, y \in \mathbb{R}, |x + y| \leq |x| + |y|$.” (20 points)

3. Use induction to prove that $\forall r \in \mathbb{R}, \forall n \in \mathbb{Z}^+, nr + 1 < (1 + r)^n$. (20 points)

4. Give a recursive definition for the sequence (a_n) defined by $\forall n \in \mathbb{N}, a_n := 3^{3^n} - 1$. (20 points)

5. Let $n \in \mathbb{Z}^+$, and $i, j \in \mathbb{Z}$. Prove that i is congruent to j modulo n if and only if n divides $i - j$. (25 points)