

Name, Last Name:

Grade:

Student No:

Math 103: Quiz # 10

Spring 2007

You have 40 minutes.

1.a) Give the statement of the Cantor-Schröder-Bernstein theorem, (10 points)

1.b) Give the definition of the sum $\alpha + \beta$ of two cardinal numbers α and β . (15 points)

2. Let \mathcal{A} be a set of sets, and \sim denote the equivalence (relation) for sets, $\mathcal{C} := \mathcal{A}/\sim$, and \preceq be defined by $\preceq := \{(\alpha, \beta) \in \mathcal{C}^2 \mid \exists A \in \alpha, \exists B \in \beta, \mathfrak{q}(A, B)\}$, where $\mathfrak{q}(A, B) :=$ “There is an everywhere-defined one-to-one function $f : A \rightarrow B$.” prove that \preceq is transitive. (25 points)

3. Prove that the set of irrational numbers is uncountable. (25 points)

4. Prove that $\aleph_0 + \aleph_0 = \aleph_0$. (25 points)