Name, Last Name: **Grade:**

Student No:

Math 103: Quiz # 1

Spring 2007

You have 45 minutes.

1. Give the definition of the following terms (No partial credit). (15 points)

1.a) Negation of a statement:

1.b) Two logically equivalent statements:

1.c) A tautology:

2. Negate the following statements. (20 points)

2.a) $\exists! x \in \mathbb{R},\ x^3 = x.$

2.b) $\forall x \in \mathbb{Z},\ \exists! y \in \mathbb{R},\ x^3 + y^5 = 2.$

3. Let $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ be statements. Show that $(\mathfrak{b} \Leftrightarrow \mathfrak{c}) \Rightarrow ((\mathfrak{a} \Rightarrow \mathfrak{b}) \Leftrightarrow (\mathfrak{a} \Rightarrow \mathfrak{c}))$ by constructing the relevant truth table. (20 points)

4. Let $\mathfrak{a}$ and $\mathfrak{b}$ be statements. Use the methods of propositional calculus to show that $\mathfrak{a} \Rightarrow \mathfrak{b} \Rightarrow \neg\mathfrak{a} \Rightarrow \neg\mathfrak{b}$ implies that $\mathfrak{a}$ is false. (20 points)

5. Let $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ be statements. Find a logically equivalent statement to $(\mathfrak{a} \Leftrightarrow \mathfrak{b}) \vee \mathfrak{c}$ that only involves $\neg$ and $\wedge$. (25 points)