# Math 103: Midterm Exam # 2

**Spring 2007**

- Write your name and Student ID number in the space provided below and sign.

| | |
|---|---|
| **Name, Last Name:** | |
| **ID Number:** | |
| **Signature:** | |

- You have 90 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

| | |
|---|---|
| **Estimated Grade:** | |

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

---

**To be filled by the grader:**

| | |
|---|---|
| **Actual Grade:** | |
| **Adjusted Grade:** | |

**Problem 1.** Give the statement of

**1.a)** Zorn's Lemma (5 points)

**1.b)** Well-ordering Axiom. (5 points)

**Problem 2.** Let $R \subseteq \mathbb{N}^2 \times \mathbb{N}^2$ be defined by $R = \{((a,b),(c,d)) \in \mathbb{N}^2 \times \mathbb{N}^2 | a+d = b+c\}$. Prove that $R$ is an equivalence relation. (15 points)
Note: You may use all the arithmetic properties of $\mathbb{N}$ without proving them.

**Problem 3.** Let $A, B, C$ be nonempty sets, $R \subseteq A \times B$ and $S \subseteq B \times C$ be relations.

**3.a)** Prove that $\mathrm{Dom}(S \circ R) = R^{-1}(\mathrm{Dom}(S))$. (15 points)

**3.b)** Prove that $\mathrm{Ran}(S \circ R) = S(\mathrm{Ran}(R))$. (10 points)

**Problem 4.** Let $(A, \preccurlyeq)$ be a poset and $B \subseteq A$.

**4.a)** Give the definition of the supremum of $B$ in $A$. (5 points)

**4.b)** Prove that the supremum of $B$ in $A$ is unique. (10 points)

**Problem 5.** Let $A, B$ be nonempty sets, $f : A \to B$ be a function, and $\{C_\gamma\}_{\gamma \in G}$ be a collection of subsets of $B$ that are labelled by the element of an indexing set $G$. Prove that

$$\bigcap_{\gamma \in G} f^{-1}(C_\gamma) \subseteq f^{-1}(\bigcap_{\gamma \in G} C_\gamma). \qquad \text{(15 points)}$$

**Problem 6.** Let $\mathbb{N}$ denote the set of natural numbers and $\mathbb{Z}^+ := \{n \in \mathbb{N} | n \neq 0\}$.

**6.a)** Prove that $\mathbb{N}$ is equivalent to $\mathbb{Z}^+$. (15 points)

**6.b)** Use 5.a to show that $\mathbb{N}$ is not a finite set. (5 points)