Math 103: Midterm Exam # 1

Spring 2007

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 105 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1.

1.a) Let $\mathfrak{a}$ and $\mathfrak{b}$ be statements and $\mathfrak{c} := (\mathfrak{a} \Leftrightarrow \mathfrak{b})$. Find a statement only involving $\neg$ and $\wedge$ that is logically equivalent to $\mathfrak{c}$. (5 points)

1.b) Let $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$ be statements, $\mathfrak{d} := (\mathfrak{a} \wedge \mathfrak{b} \wedge \mathfrak{c})$ and $\mathfrak{g} := ((\mathfrak{a} \wedge (\mathfrak{a} \Rightarrow \mathfrak{b} \Rightarrow \mathfrak{c})))$. Show that $\mathfrak{d} \Leftrightarrow \mathfrak{g}$ is a tautology. (5 points)

Problem 2.

2.a) Prove that $\forall p \in \mathbb{Z},\ 3|p^2 \ \Rightarrow\ 3|p.$ (10 points)

2.b) Prove that $\sqrt{3}$ is not a rational number. (15 points)

Problem 3. Let $\phi := \frac{1}{2}(1+\sqrt{5})$ and f_n denote the the Fibonacci numbers, i.e., $f_1 := 1$, $f_2 := 1$, and for all $n \geq 2$, $f_{n+1} := f_n + f_{n-1}$. Use complete induction to prove the following statement. (15 points)

$$\forall n \in \mathbb{Z}^+, \quad f_n = \frac{1}{\sqrt{5}}\left[\phi^n - (-1)^n \phi^{-n}\right].$$

Hint: You may use the following following identities: $\phi - 1 = \phi^{-1}$ and $\phi + 1 = \phi^2$.

Problem 4. Let A, B, and C be sets.

4.a) Prove that $A \setminus (A \setminus B) = A \cap B$. (10 points)

4.b) Prove that $(A \subseteq C) \Rightarrow (\mathcal{P}(A) \subseteq \mathcal{P}(C))$. (10 points)

Problem 5.

5.a) Let A and B be sets, $a \in A$, and $b \in B$. Give the definitions of $\{a\}$ and $\{a, b\}$, and prove that they are sets. (5 points)

5.b) Give the statement of the Axiom of Pairing. (5 points)

5.c) Give the definition of the ordered pair (a, b) and prove that it is a set. (5 points)

5.d) Give the definition of $A \times B$ and prove that it is a set. (5 points)

Problem 6. Give the definition of addition and multiplication of natural numbers and use them to prove that

6.a) $3 + 2 = 5$. (5 points)

6.b) $3 \cdot 2 = 6$. (5 points)