Solutions

Math 103: Quiz # 6

Fall 2007

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 50 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Give the definition of the following terms. See the book.

1.a) well-defined relation: (5 points)

1.b) one-to-one function: (5 points)

1.c) onto function: (5 points)

1.d) bijection: (5 points)

2. Let A, B, C be sets and $f : A \to B$ and $g : B \to C$ be functions such that $\mathrm{Ran}(f) \cap \mathrm{Dom}(g) \neq \varnothing$. Prove the following statements.

See the book

2.a) $g \circ f : A \to C$ is well-defined. (5 points)

2.b) If both f and g are everywhere-defined, then $g \circ f : A \to C$ is everywhere-defined. (5 points)

2.c) If both f and g are one-to-one, then $g \circ f : A \to C$ is one-to-one. (5 points)

2.d) If both f and g are onto, then $g \circ f : A \to C$ is onto. (5 points)

3. Let A, B be nonempty sets and $f : A \to B$ be an everywhere-defined function. Show that $R := \{(x, y) \in A^2 | f(x) = f(y)\}$ is an equivalence relation. (30 points)

- $\forall x \in A$, $x \in \mathrm{Dom}(f)$ because f is everywhere defined $\Rightarrow$ $\exists y \in A$, $y = f(x)$. y is unique because f is a function. $\Rightarrow$ $f(x) = f(x)$ $\Rightarrow$ $x R x$ $\Rightarrow$ R is reflexive. (1)

- $\forall x, y \in A$, $xRy \Rightarrow f(x) = f(y) \Rightarrow f(y) = f(x) \Rightarrow y R x$ $\Rightarrow$ R is symmetric. (2)

- $\forall x, y, z \in A$, $(xRy) \wedge (yRz) \Rightarrow (f(x) = f(y)) \wedge (f(y) = f(z))$ $\Rightarrow f(x) = f(z) \Rightarrow xRz \Rightarrow$ R is transitive. (3)

(1), (2), (3) $\Rightarrow$ R is an equivalence relation.

4. Let A, B be nonempty sets, $f : A \to B$ be an everywhere-defined 1-to-1 function, and $S \subseteq B^2$ be a partial ordering relation. Show that $R := \{(x, y) \in A^2 | \ f(x)\ S\ f(y)\ \}$ is a partial ordering relation. (30 points)

See Thm 6-2.2 of the book