Math 103: Midterm Exam 2

Fall 2007

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have two hours.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1.

1.a) Give the definition of an inductive set. (2 points)

1.b) Give the definition of the set $\mathbb{N}$ of natural numbers. (2 points)

1.c) Prove that $\mathbb{N}$ is an inductive set. (6 points)

Problem 2. Let A, B, C be nonempty sets, $X \subseteq A \times B$ and $Y \subseteq B \times C$ be relations. Prove that $(Y \circ X)^{-1} = X^{-1} \circ Y^{-1}$. (15 points)

Problem 3. Let S be a nonempty set and $R \subseteq S^2$ be a reflexive and transitive relation.

3.1) Prove that $E := \{(x,y) \in S^2 \mid (x \ R \ y) \wedge (y \ R \ x) \}$ is an equivalence relation. (10 points)

3.2) Prove that $P := \{(A,B) \in (S/E)^2 \mid \exists \ a \in A, \ \exists b \in B, \ a \ R \ b \}$ is a partial ordering relation. (10 points)

Problem 4. Let A, B be sets, $C \subseteq A$, C^c be the complement of C in A, and $f : A \to B$ be a one-to-one function. Prove that $f(C^c) = \text{Ran}(f) \setminus f(C)$. (15 points)

Problem 5. Let $(A, \preccurlyeq)$ be a poset and $s : \mathbb{Z}^+ \to A$ be a sequence in A.

5.1) Give the definition of a subsequence of s. (5 points)

5.2) Prove that if s is an increasing sequence, every subsequence of s is also an increasing sequence. (10 points)

Problem 6. Let A and B be finite sets. Prove that A is equivalent to B if and only if $\text{Ord}(A) = \text{Ord}(B)$ (10 points)

Problem 7. Let $A := \{n \in \mathbb{N} \mid \exists m \in \mathbb{N},\ n = m^2\ \}$. Prove that A is an infinite set. (15 points)