Math 103: Midterm Exam 1

Fall 2007

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have One hour and 45 minutes (105 minutes).
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$ be statements and

$$\mathfrak{t} := (((\mathfrak{b} \Rightarrow \mathfrak{a}) \Rightarrow (\mathfrak{b} \wedge \mathfrak{c})) \Leftrightarrow (\mathfrak{b} \wedge (\mathfrak{a} \Rightarrow \mathfrak{c}))).$$

Use the methods of propositional calculus to prove that $\mathfrak{t}$ is a tautology. (10 points)

Problem 2. Let $n \in \mathbb{Z}^+$, $k_1, k_2, r_1, r_2 \in \mathbb{Z}$, and $0 \leq r_1 < n$ and $0 \leq r_2 < n$. Prove the following statements without using the division algorithm. You are actually asked to prove the uniqueness part of the division algorithm.

2.a) "n divides $|r_1 - r_2|$" implies "$r_1 = r_2$". (10 points)

2.b) "$k_1 n + r_1 = k_2 n + r_2$" implies "$(r_1 = r_2) \wedge (k_1 = k_2)$". (5 points)

Problem 3.

3.a) Give the statement of the Induction Axiom. (2 points)

3.b) Give the statement of the Principle of Mathematical Induction. (2 points)

3.c) Use the Induction Axiom to prove the statement of the Principle of Mathematical Induction. (6 points)

Problem 4. Use induction to prove that $\forall n \in \mathbb{N},\ 7 \Big| (2^{n+2} + 3^{2n+1})$. (15 points)

Problem 5. Let for any set S the power set of S be denoted by $\mathcal{P}(S)$ and

$$\mathfrak{a} := \text{“For any pair of sets } A \text{ and } B,\ \mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B).\text{”}$$
$$\mathfrak{b} := \text{“For any pair of sets } A \text{ and } B,\ \mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B).\text{”}$$

5.a) Prove that $\mathfrak{a}$ is true. (10 points)

5.b) Prove that $\mathfrak{b}$ is false. (10 points)

Problem 6. Let $\mathfrak{c}$:=“for any collection $\{A_\beta\}_{\beta\in\mathfrak{B}}$ of sets labelled by the elements of an indexing set $\mathfrak{B}$ there exists a universal set for the members of $\{A_\beta\}_{\beta\in\mathfrak{B}}$.”

6.a) Give the statement of the Union Axiom. (5 points)

6.b) Prove that the Union Axiom is logically equivalent to $\mathfrak{c}$.. (10 points)

Problem 7. Let S be a set and $\{A_\beta\}_{\beta\in\mathfrak{B}}$ be a collection of sets labelled by the elements of an indexing set $\mathfrak{B}$. Prove that

$$S \times \big(\bigcap_{\beta\in\mathfrak{B}} A_\beta\big) = \bigcap_{\beta\in\mathfrak{B}} (S \times A_\beta). \qquad \text{(15 points)}$$