

## Elec 206/Phys 302

### Homework Assignment 1

Due on Thursday, October 7, 2021, at 22:00

1 (20 pts) Use the identity  $\sum_{i=1}^3 \epsilon_{ijk} \epsilon_{i\ell m} = \delta_{j\ell} \delta_{km} - \delta_{jm} \delta_{k\ell}$  to show that for every

$\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$ ,

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = \mathbf{0}.$$

2 (20 pts) Calculate the gradient and Laplacian of the function,  $f(\mathbf{r}) := r^n$ , where  $\mathbf{r} := (x, y, z)$ ,  $r := |\mathbf{r}|$ , and  $n$  is a positive or negative integer.

3 (15 pts) Calculate the divergence of the vector-valued function,  $\mathbf{A}(\mathbf{r}) := \frac{\mathbf{r} - \mathbf{a}}{|\mathbf{r} - \mathbf{a}|^n}$ ,

where  $\mathbf{r} := (x, y, z)$  and  $\mathbf{a}$  is a constant vector.

4 (45 pts) Establish the following identities for every scalar function  $f(\mathbf{r})$  and vector-valued functions  $\mathbf{A}(\mathbf{r})$  and  $\mathbf{B}(\mathbf{r})$ , where  $\mathbf{r} := (x, y, z)$

4.a)  $\nabla \cdot [f(\mathbf{r})\mathbf{A}(\mathbf{r})] = \nabla f(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r}) + f(\mathbf{r}) \nabla \cdot \mathbf{A}(\mathbf{r})$

4.b)  $\nabla \times [f(\mathbf{r})\mathbf{A}(\mathbf{r})] = \nabla f(\mathbf{r}) \times \mathbf{A}(\mathbf{r}) + f(\mathbf{r}) \nabla \times \mathbf{A}(\mathbf{r})$

4.c)  $\nabla \cdot [\mathbf{A}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})] = \mathbf{B}(\mathbf{r}) \cdot [\nabla \times \mathbf{A}(\mathbf{r})] - \mathbf{A}(\mathbf{r}) \cdot [\nabla \times \mathbf{B}(\mathbf{r})]$