Zeta-functions and arithmetic: From Euler to Wiles

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The purpose of this expository talk is to discuss an important theme in Number Theory: The relation between zeta-functions (objects of analytic nature) and certain objects (which we generally call them Selmer groups) of arithmetic nature. Kummer was first to recognize the arithmetic significance of the special values of the classical zeta-function, using which he was able to deduce an important portion of the "Fermat's Last Theorem". Kummer's ideas were much later generalized by Ribet and Wiles (in a certain sense) to conclude with the full proof. An important portion of this talk will be devoted to explaining Kummer's ideas, and if time permits, say a few words about their influence in modern number theory (e.g., towards Birch-Swinnerton Dyer conjectures (one of the Clay Millenium Problems) and its utmost generalization, the Bloch-Kato conjectures).