The Wilkinson Distance of a Matrix

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The eigenvalues of a matrix associated with a Jordan block of size two or greater (such a block is called a defective Jordan block) are highly sensitive to perturbations. This implies that all matrices close to a non-diagonalizable matrix have ill-conditioned eigenvalues. In his book *the algebraic eigenvalue problem* J.H. Wilkinson points out that a defective Jordan block is one source of ill-conditioned eigenvalues and another is being close to a matrix with a defective Jordan block. The Wilkinson distance of a matrix is the distance in 2-norm to the nearest matrix with a defective Jordan block or equivalently with a multiple eigenvalue. We derive a singular value optimization characterization for a generalized version, the distance to the nearest matrix with an eigenvalue of algebraic multiplicity r, where r is given *a priori*. The singular value optimization characterization derived makes the computation of the generalized Wilkinson distance feasible. At the end of the talk we will point out the rather surprising resemblance of a model reduction problem in control theory to the Wilkinson distance.