ALGORITHMS FOR APPROXIMATING THE H_∞ NORM

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H_∞ norm methods are used in control theory to design optimal controllers. The controllers are designed so as to minimize the H_∞ norm of the $n \times n$ closed-loop transfer matrix where n is the system order. This optimization procedure necessitates efficient methods for the computation of the H_∞ norm itself. Existing methods compute H_∞ norm accurately but the cost is multiple singular value decompositions or eigenvalue decompositions of size n, making them impractical when n is large. We present a novel method which provides a fast computation of the H_∞ norm for large and sparse matrices, such as the ones arising in the control of PDE's. The method is a nested fixed point iteration, where the outer iteration is a Newton step and the inner iteration is associated with the problem of the computation of the ε-pseudospectral abscissa, i.e. the real part of a righmost point of the ε-pseudospectrum of a linear operator in a sense to be precised. We characterize fixed points of the iteration, proving a locally linear rate of convergence for small enough ε. We will give some applications to the control of PDE's. We also prove some regularity results about ε-pseudospectral abscissa, including global Lipschitzness, which led to a resolution of the Lewis-Pang conjecture. If time permits, I will describe some of our other work on fixed-order controller design. This is joint work with Michael Overton and Nicola Guglielmi.