

Invariant Measures Concentrated on Countable Structures

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The Erdos-Renyi random graph construction can be seen as inducing a probability measure concentrated on the Rado graph (sometimes known as the countable “random graph”) that is invariant under arbitrary permutations of the underlying set of vertices. The following question arises naturally:

On which countable structures is there such an invariant measure?

Up until recent work of Petrov and Vershik (2010), it was not even known if Henson’s universal countable triangle-free graph admitted an invariant measure.

We provide a complete characterization of countable structures admitting invariant measures in terms of the model-theoretic notion of “definable closure”. This will lead to a classification of those ultrahomogeneous graphs which admit invariant measures as well as allow us to show the existence of invariant measures on many other structures.