Resolutions of t-designs, orthogonality, and group-theoretic constructions

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Resolutions of $t-$designs were studied as early as 1847 by Reverend T. P. Kirkman [2, 3] who proposed the famous 15 schoolgirls problem (see also [1]). Kirkman's problem is equivalent to finding a *resolvable* 2-$(15,3,1)$ design with $r = 7$, and $b = 35$. We define and discuss τ-*resolutions* of t-designs, *large sets* of t-designs, *orthogonal resolutions* of t-designs, *Room rectangles* [4, 5], and *Steiner tableaux*. We briefly discuss recursive constructions of large sets, but spend more time on techniques for constructing *starter* large sets which can then be used in the recursive techniques to obtain infinite families of large sets. We pay particular attention to *coherence* techniques, i.e. construction methods which assume particular automorphism groups under which the above combinatorial objects are invariant. We give examples of large sets and *super-large* sets constructed by means of groups. We discuss infinite families of *semiregular* large sets arising from 3-homogeneous actions of the groups $PSL_2(q)$ on the projective line, and time permitting, we discuss the construction of certain sporadic, as well as infinite families of Room rectangles arising from block-transitive but imprimitive group actions. Finally, we present some tantalizing open problems.

References

[1] A. Cayley. On the triadic arrangements of seven and fifteen things. *London, Edinburgh and Dublin Philos. Mag. and J. Sci.*, **37** (1850) 50–53.

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[3] T. P. Kirkman. Note on an unanswered prize question. *Cambridge and Dublin Math. Journal*, **5** (1850), 255–262.

[4] E.S. Kramer, D.L. Kreher, S.S. Magliveras and D.M. Mesner, Coherent Room Rectangles from Permutation Groups, *Ars Combinatoria,* **9** (1981), pp 101-111.

[5] E.S. Kramer, S.S. Magliveras, D.M. Mesner, An Assortment of Room-type Designs, *Ars Combinatoria,* **11** (1981), pp 9-29.

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