Combinatorial t-Restrictions

Charles J. Colbourn
Computing, Informatics, and Decision Systems Engineering
Arizona State University
PO Box 878809, Tempe, AZ 85287-8809, U.S.A.

Many fundamental combinatorial problems can be expressed in the following framework. Let Σ be a set of symbols, which may be finite or infinite. Let $t > 0$ be an integer. Let A be an $m \times t$ matrix with symbols from Σ. For $X \subseteq \Sigma^t$, the integer variable $\rho(A, X)$ counts the rows of A that appear in X. A *basic t-restriction* on A is a requirement that $\rho(A, X)$ be at least, or at most, or exactly, a specified constant. Then a *t-restriction P* on A is a logical formula whose terms are basic t-restrictions. For a matrix M that is $m \times k$ for $k \geq t$, M *satisfies* P when for every way to select t columns of M (in order), the $m \times t$ submatrix so formed satisfies P.

A typical combinatorial example is a *pairwise balanced design*; take $\Sigma = \{0, 1\}$, and treat the $b \times v$ incidence matrix of blocks against points. Then the 2-restriction satisfied is that there is an integer λ so that $\rho(A, (1, 1)) = \lambda$ for every $b \times 2$ submatrix A of the incidence matrix. Standard combinatorial designs, such as balanced incomplete block designs, t-designs, packings, and coverings all fit into this framework. When the alphabet is larger, we encounter orthogonal arrays, covering arrays, and error-correcting codes ($\equiv$ packing arrays). Moreover, many different types of so-called hash families are obtained by considering larger subsets for X in the basic restrictions.

In this talk, we use the framework of t-restrictions to discuss a general recursive construction for combinatorial matrices.